Funding Shocks and Optimal University Admissions and Financial Aid Policies

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Abstract

A positive shock to funding, such as a major donation, causes an optimizing university to raise its admissions standards and reduce tuition charges net of financial aid across all student categories. However, the shock's effect on enrollment may not be uniform. Student categories given little weight in the university's objective function may be treated as "inferior goods," that is, positive shocks decrease their enrollments, while other student categories' enrollments are increased. The paper's findings shed light on the effect of federal direct-to-student aid on tuition levels, permitting a new perspective on William Bennett's controversial hypothesis that aid accommodates tuition hikes. *(JEL I22, L31)*

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Introduction

On August 1, 2006, Stanford University announced that Philip H. Knight, founder and chairman of Nike, Inc., would give $105 million to its Graduate School of Business ("GSB").[1] The event provides a stark example of a funding shock – a major change, either positive or negative, in the level of funding available from endowment and external sources for university expenditures. Other examples would include receiving a major research grant, experiencing a sharp drop in the value of endowment investments, or experiencing a cut in state appropriations.

A funding shock may be a highly significant event for an institution of higher learning. The endowment of Stanford GSB was valued at $711.8 in 2005, so Knight's donation increased it by about 15%.[2] Events of this magnitude may affect financial aid, admissions, and enrollment decisions in important ways.[3] Given the recent prevalence of both record donations to universities and colleges and ongoing cutbacks in federal and state funding, seeking an improved understanding of the impacts of funding shocks would seem warranted.

This paper employs a theoretical model to examine the effect of a funding shock on optimal university decision-making with respect to the level of tuition charged to students, net of financial aid ("net tuition"); admissions selectivity; and enrollment. The model considers a situation in which different student groups are characterized by different levels of relative attractiveness to the university, independent of objectively measurable quality.[4] Enrolled students with greater relative attractiveness carry greater weight in the university's utility function; thus the university cares more both about enrolling more attractive students and about their relative quality when enrolled.

A key recognition of the model is that students whose net tuition is set high relative to their costs of enrollment, whether because their relative attractiveness to the university is low or because the university's resources are limited, are treated differently from other students by an optimizing university when a funding shock occurs. I find that a positive shock, such as a major donation, lowers net tuition to all students, a result consistent with existing empirical work [Lowry, 2001;

Rizzo & Ehrenberg, 2004]. Not surprisingly, it also increases the university's admissions selectivity with respect to all students, as the university balances competing goals of higher quantity and quality of students. However, a positive shock results in *lower* enrollment for students whose net tuition is high relative to cost, while resulting in *higher* enrollment for all others. For this reason, I refer to high-margin students as "inferior goods" to the university, as the effect of additional funding on the university's enrollment of these students is analogous to the effect of additional income on the quantity an individual consumes of an inferior good.

There has been some recognition in the literature that certain student groups may be singled out by universities for use as revenue generators. Mixon & Hsing [1994] observe that public universities view out-of-state students as a source of revenue given that they typically pay higher tuition. The authors even point out the importance to states of in-migration of students in light of budget cuts in higher education. Balderston [1997] offers anecdotal evidence from the University of Michigan of the use of out-of-state enrollment to increase revenue. However, it does not appear that anyone has systematically studied under what conditions a student group will be targeted in this way.

Beyond shedding light on the effect of funding shocks *per se*, the model offers a comparative static framework that can be extended to analyze the effects of various changes in market conditions. To illustrate, I use the model to analyze the effect on net tuition of a shock to direct-to-student financial aid. William Bennett, President Reagan's education secretary, asserted in the mid-1980s that federal financial aid programs "enable" universities to increase tuition by allowing them to rely on aid to cushion the effects of increases [Bennett, 1987]. Bennett's hypothesis has received considerable scholarly attention,[5] but as yet no clear consensus has emerged concerning its validity.

The next section reviews the related literature. In the sections that follow, I describe the model and present its main results relating to the effects of funding shocks. The effect of an increase in federal direct-to-student aid is then considered. The final section concludes.

Related Literature

Reflecting the generally-accepted notion that institutions of higher learning are not for-profit enterprises, a substantial theoretical literature uses utility-maximization models to describe university behavior. Garvin [1980] and James [1990] offer detailed models in which the university's objective is to maximize some function of the quantity and quality of students and faculty (or, equivalently, research), subject to a resource constraint. Using a framework more closely resembling that in the present paper, Ehrenberg & Sherman [1984] abstract from issues relating to faculty and administrative inputs and focus on the student side of the university's problem,. They consider how optimal financial aid packages should be set, given varying student demand characteristics and variations in relative university preferences across student groups. They draw important conclusions, but do not address how changes in funding, as reflected in the university's resource constraint, should affect financial aid decision-making.

Two other theoretical analyses closely related to the present paper are offered by Fethke [2005, 2006], who uses a principal-agent framework to study the relationship of state appropriations to public university tuition levels in a context in which states compete. Fethke [2005] performs a positive economic analysis on this basic framework, while Fethke [2006] focuses on normative considerations, deriving vertical coordination strategies for arriving at optimal subsidy and tuition levels. The latter paper also derives two descriptive conclusions that extend across the alternative strategic arrangements considered: first, that the difference between non-resident and resident tuition declines with decreases in state appropriations, and second, that it declines with increases in the demand for or cost of education. An important characteristic of Fethke's framework is that he assumes state legislatures set subsidies strategically, recognizing the effect that they have on university tuition-setting. Thus, he endogenizes the subsidies in the tuition model. In contrast, I focus on the effects of exogenous shocks to university funding; this means, to the extent that my model applies to state appropriations, it assumes them to be exogenously determined.

Also relevant to the present paper are several empirical studies that have considered the effects of state appropriations on tuition and enrollment at public universities [Koshal & Koshal, 2000; Cunningham et al., 2001; Lowry, 2001; Rizzo & Ehrenberg, 2004]. These studies offer insight into some of the ramifications of funding shocks by reference to an important example of such shocks. But no work has yet provided a general conceptual understanding of how funding shocks affect university decisions or how they differentially affect different groups of students.

A Model

Consider a university enrolling students from two groups, $i = 1, 2$. The groups may be thought of as consisting of athletes and non-athletes, state residents and non-resident students, poor and wealthy students, or any other appropriate dichotomy. The university charges "net tuition," P_i, to group *i*, where net tuition consists of a positive level of full tuition minus a nonnegative amount of institutional financial aid. Thus, net tuition may differ for the two groups either because full tuition differs, financial aid packages differ, or both.[6]

The net tuition level P_i generates a demand for admission (i.e., number of applicants), $Q_i\left(P_i\right)$. The groups' demands are mutually independent and are given by a general functional form [Genesove and Mullin, 1998],

$$Q_i\left(P_i\right) = b_i\left(a_i - P_i\right)^{\gamma_i} \tag{1}$$

where $a_i > 0$ is the student's maximum willingness to pay, $b_i > 0$ represents the size of the market, and $\gamma_i > 0$ is an index of convexity. Note that linear ($\gamma_i = 1$) and quadratic ($\gamma_i = 2$) demand curves, among other forms, are nested as special cases. The university is selective, that is, it does not enroll every student who applies. Rather, it accepts a share $0 \le \phi_i \le 1$ of the applicants from group *i*. I assume that all accepted students enroll at the university, so enrollment E_i is defined $E_i \equiv \phi_i Q_i$.

The university obtains utility from enrolling students. Its values are summarized by a weighted-enrollment utility function,

$$U = \alpha h_1(\phi_1) E_1(\phi_1, P_1) + (1-\alpha) h_2(\phi_2) E_2(\phi_2, P_2) \tag{2}$$

Here, $h_i(\phi_i)$ is the *average quality level* of accepted applicants in group *i*, and the weight $0 \le \alpha \le 1$ reflects the *relative attractiveness* of the two groups to university. One may think of h_i as reflecting a composite of objective measures of academic quality, such as test scores and grade-point average. Meanwhile, α reflects the level of subjective preference on the part of university administrators for group 1 relative to group 2. Let h_i be defined as a linear function of ϕ_i,

$$h_i(\phi_i) = \mu_i + \sigma_i \phi_i \tag{3}$$

where $\mu_i > 0$ and $0 > \sigma_i > -\mu_i/\phi_i$. Thus, h_i is a decreasing function of the share of applicants accepted, and is positive and bounded on its domain.[7]

The university utility function posited here is based on generally-endorsed notions about university behavior, as well as constructs previously proposed in the literature. The average-quality-weighted-enrollment form of the function reflects the idea that universities value both the quantity and academic quality of enrolled students.[8] The use of subjective, non-quality-related group weights is adapted from Ehrenberg and Sherman (1984) and allows for the incorporation of administrator preferences apart from those dictated by observable quality measures.[9] The chief issue with respect to the generality of the posited function is that the average student quality in each group *i* has been assumed to be a linear function of the selectivity parameter, ϕ_i. I maintain this assumption for clarity of exposition; however, the paper's results are generally robust with respecting to relaxing it.[10]

The university's objective is to choose net tuition levels P_i and selectivity levels ϕ_i to maximize (2) subject to breaking even on its operating account. This break-even requirement is summarized by the following resource constraint

$$R \equiv (c_1 - P_1) E_1(\phi_1, P_1) + (c_2 - P_2) E_2(\phi_2, P_2) - F \leq 0 \quad (4)$$

where $c_i > 0$ represents the marginal cost of enrolling students from group *i*,[11] and F, which may be positive or negative, represents funds available from the endowment, government appropriations, and other non-student sources. In words, (4) states that the cost of enrolling students, after net tuition is subtracted out, must not exceed the funds available from non-student sources.

In the context of this framework, a change in F will be termed a *funding shock*. Note that a positive funding shock is equivalent to a relaxation of the resource constraint, while a negative shock equates to a tightening of the constraint. Differentiating the expression for enrollment with respect to F and evaluating at the optimizing (ϕ_i^*, P_i^*) pair gives:

$$\frac{\partial E_i}{\partial F} = \phi_i^* \frac{\partial Q_i}{\partial P_i} \frac{\partial P_i^*}{\partial F} + Q_i \frac{\partial \phi_i^*}{\partial F} \quad (5)$$

When the expression in (5) is positive, a positive shock to funding increases the equilibrium enrollment of group *i*. This is intuitively what one would expect to happen to enrollment when more money is available to the university; after all, (2) shows that the university obtains utility from enrolling students. But when this expression is negative, a positive shock to funding decreases the enrollment of group *i*. This would seem to represent an anomaly. Why would a university cut back the number of students it enrolls from a group when it has more money with which to subsidize enrollment? Paralleling the terminology of consumer theory, I will refer to a group of students *i* for whom $\frac{\partial E_i}{\partial F} > 0$ as a "normal good" to the university. A group of students for whom $\frac{\partial E_i}{\partial F} < 0$ will be referred to as an "inferior good." The main focus of the next section will be to explain when and why the anomalous outcome – inferiority – occurs.

Results

The sign of $\frac{\partial E_i}{\partial F}$ depends on the signs of $\frac{\partial P_i^*}{\partial F}$ and $\frac{\partial \phi_i^*}{\partial F}$, as well as the relative size of the two terms in (5). Comparative static techniques may be used to make these determinations. Our first result relates to the signs of $\frac{\partial P_i^*}{\partial F}$ and $\frac{\partial \phi_i^*}{\partial F}$ (proofs of all propositions and corollaries are provided in the Appendix):[12]

Proposition 1: Negative funding shocks should result in higher net tuition for all groups and reduced admissions selectivity with respect to all groups.

By lowering standards, the university is able to admit more students, thus offsetting somewhat the reduction in enrollment that occurs from raising net tuition. Though doing so means the university must accept some reduction in average quality, it optimally balances this cost against the cost of reducing enrollment. The new balance that is struck following a reduction in funding involves some increase in net tuition and some reduction in selectivity.

Before turning to the conditions for inferiority, it is helpful to understand the determinants of the sign of the tuition-cost margin, $P_i - c_i$. These are laid out in the following proposition:

Proposition 2: Students should be charged net tuition greater than their marginal cost of enrollment if their group's relative attractiveness to the university is low enough or if endowment and other non-tuition funding sources are sufficiently limited.

The proposition formalizes the conditions that result in a group of students cross-subsidizing other activities of the university.[13] Not surprisingly, if a group is of very low relative attractiveness to the university, its tuition revenues will be used to fund the enrollment of more attractive groups. But a student group may end up subsidizing university operations even if it does not exhibit low relative attractiveness if the university is truly impoverished.

The following proposition establishes an important result relating to the $P_i > c_i$ case:

Proposition 3: When a group is charged net tuition in excess of its marginal cost of enrollment, the university admits from the group some students that are of such low quality that they make a negative contribution to the university's utility.

The university admits students from a group until the utility contributed by the marginal student just equals the net monetary cost of enrolling that student. Since $P_i > c_i$, the university incurs a *negative* net cost (i.e., earns a positive return) from enrolling students. This means that the marginal student will provide the university with negative utility.

One way to think about the meaning of this is to think of a university's student body as consisting of two types of students: prestige generators and revenue generators. On the one hand, a selective university desires students that reflect well on the institution, and it will often be willing to pay handsomely to have such students, offering them lavish financial aid packages. On the other hand, the university may be willing to take on some real "undesirables" from a prestige perspective, if it gets sufficient money to do so. So, though we might think of the former type of student as "favored" and the latter "disfavored" by the university, it is perhaps truer to say that the university wants both types of student around, but for different reasons.

Now let us return to the question of inferiority. The following proposition offers the main result with respect to the effect of a funding shock on equilibrium enrollment:

Proposition 4 (Inferiority Condition): Negative funding shocks reduce the equilibrium enrollment level for all groups, except for groups for whom $\frac{P_1 - c_1}{P_1} > \frac{\gamma_1}{\gamma_1 + 1}\frac{1}{\varepsilon_1}$, where ε_i is group *i*'s demand elasticity. For these groups, negative shocks should result in an increase in enrollment.

That is, when a group's tuition-cost margin is sufficiently elevated given demand elasticity (and the extent of demand convexity), funding cuts should actually cause the group's enrollment to increase. In effect, when more money is needed, the university manipulates net tuition and selectivity to increase the enrollment of groups whose members generate sufficient revenue. Note that the higher the demand elasticity, the lower the threshold for inferiority (i.e., closer to $P_i = c_i$). Intuitively, a university will raise net tuition less in the face of funding cuts for groups exhibiting greater demand elasticity, making it more likely that their enrollment will increase rather than decrease.

This finding leads to an important general prediction: the optimizing university's student population will shift in favor of student groups that cover their costs when money is tight. Conversely, it will generally shift in favor of groups that depend on subsidization when money is easy. Put another way, the composition of a student body can be expected to shift with the university's financial fortunes, alternating between prestige-generating students and revenue-generating students.

While Proposition 4 has intuitive appeal in that it makes a connection between inferior good status and a group's tuition-cost margin, it would also be useful to connect inferiority with exogenous group characteristics. Hence, the following corollary is offered:[14]

Corollary 1: An optimizing university will increase a group's enrollment in the face of a negative funding shock if the group exhibits sufficiently low relative attractiveness to the university, or if endowment and other non-tuition funding sources are sufficiently limited.

Note that if a university's resources are sufficiently constrained, all its students could be inferior goods. Funding cuts would lead to increased enrollment for all, while increases in non-tuition funding would lead to a general reduction in enrollment. This may seem topsy-turvy, but recall that when a group is charged net tuition greater than its marginal cost of enrollment, some students are enrolled who make a negative contribution to the university's utility. Reducing enrollment for such groups by increasing admissions standards actually increases utility. Thus, "weeding out" undesirable students from a body of applicants may be thought of as a luxury that the resource-constrained university enjoys when it receives a positive shock to its funding.

A stylized fact that follows from Proposition 1 is that, for normal good students, the enrollment effect of changes in net tuition brought about by funding shocks dominates the enrollment effect of changes in selectivity, while for inferior good students, the opposite is true. Thus:

Proposition 5: Given equivalent demand for admission, Q_i, and equivalent responsiveness of enrollment to changes in net tuition, $\phi_i \frac{\partial Q_i}{\partial P_i}$, for both types of groups, normal good student groups experience greater volatility in net tuition and lower volatility in admissions standards in response to funding shocks than inferior good student groups.

All else being equal, inferior good students' average quality will tend to be highly variable in response to funding shocks. Meanwhile, these students will not tend to suffer large fluctuations in their cost of attending as a consequence of such shocks. However, normal good students will suffer such fluctuations, while their average quality level will tend to be more stable.

Effect of an Increase in Federal Direct-to-Student Aid

In this section, I use the framework introduced above to investigate William Bennett's contention that federal direct-to-student aid creates incentives for universities to raise their tuition levels. Analysts have provided two main rationalizations of Bennett's hypothesis.[15] First, properlytargeted aid shifts outward the demand curve for higher education by increasing the number of individuals that can afford to attend. This, it is argued, motivates institutions to increase tuition by making such increases more profitable. Second, federal aid programs create direct incentives for raising tuition by making aid awards dependent upon student need, where "need" is defined in part based on the size of tuition.

To this day, Bennett's hypothesis remains a topic of intense debate. A number of studies have tried to discern whether federal direct aid leads to increased tuition, and if so, under what circumstances this occurs. Hauptman & Krop [1998] point to trend data as evidence that the increased availability of federally guaranteed loans has accommodated increases in tuition. Analyzing a simultaneous equations model, McPherson & Schapiro [1991] find no evidence of a relationship between federal grant aid and tuition increases for private institutions, but they do find a significant positive relationship for public 4-year institutions. They attribute this to public institutions having low enough tuition levels that raising them could affect student qualification for major federal award programs, such as the Pell grant or Guaranteed Student Loan. Turner [1998] considers the extent to which benefits of the Pell grant program flow to its intended beneficiaries. Her theoretical analysis suggests that universities already giving substantial institutional aid to needy students may seize on Pell grants as an opportunity to free up those funds for other uses, thus partially diverting Pell program benefits. Estimating a series of reduced-form models, Cunningham et al. [2001] find no relationship between any form of external grant aid, including federal aid, and changes in tuition at any of seven higher-education institution types.

In the wake of this work, one conceptual issue that remains open is whether the increase in demand brought about by a positive shock to federal aid would indeed lead to an increase in tuition. In the model introduced in Section 3, a shift in demand may be represented by a change in b_i in (1). It can be shown that the effect of an increase in demand on net tuition is proportional to the effect of a positive funding shock on net tuition, where the proportionality factor is $-(c_i - P_i)\phi_i(a_i - P_i)^{\gamma_i}$.[16] Proposition 1 establishes that a positive funding shock results in decreased net tuition, so the effect of an increase in demand takes the sign of $c_i - P_i$. In words, an increase in demand for education results in higher net tuition when net tuition is less than marginal cost, and lower net tuition when net tuition is greater than marginal cost. Intuitively, an increase in demand causes the university to lower net tuition if the increase generates revenue for the university (by increasing the enrollment of revenue-generating students), while it causes the university to raise net tuition if it drains revenue away (by increasing the enrollment of subsidized, prestige-generating students).[17]

Consider the implications of this for the Bennett hypothesis. First, an increase in demand does not necessarily lead to increased tuition at a utility-maximizing university the way it would at a profit-maximizing firm. Thus, Bennett may have based his hypothesis, at least in part, on an incorrect implicit assumption of profit maximization. Second, caution should be exercised in interpreting McPherson & Schapiro's [1991] differing results for public and private institutions as indicating that public universities are trying to game the structure of the Pell program. Given a mission to provide an affordable education to resident students, most public universities likely have a greater percentage of students paying net tuition below marginal cost than private universities. Thus, public institutions might see average tuition levels increase with increases in federal aid because the average student's net tuition is below cost. Meanwhile, private institutions might not see an increase in their average tuition level because the average student pays net tuition close to or above cost.

Conclusion

At an inn visited by the author while researching this paper hangs a sign that reads, "All visitors bring us happiness – some when they arrive, and some when they leave." In perhaps a similar spirit, as this paper has suggested, all students benefit the university – some by bringing prestige or an uplifting sense of having served some community of interest, and some by bringing money. The model developed in the paper has demonstrated that, in the face of a negative shock to university funding, the optimizing university reduces enrollment of the former type of student, while increasing enrollment of the latter. The opposite occurs when there is a positive shock to funding. The university's "favored" students (those bringing prestige) experience greater volatility in net tuition than the "disfavored" students (those bringing money), while the disfavored students experience greater volatility in admissions standards than the favored students. This makes it possible for favored and disfavored student enrollments to be negatively correlated in the face of a funding shock, while both groups of students experience the same direction of movement of net tuition and admissions selectivity.

The model's results were applied to analyzing the question of whether increases to federal direct-to-student aid lead universities to increase tuition. It was found that, for a utility-maximizing institution, an increase in tuition would not necessarily follow from the increase in demand that a positive shock to aid would cause. The particular pattern of tuition increases predicted by the model is consistent with empirical outcomes previously attributed to public universities strategically increasing tuition to take advantage of the structure of federal aid programs. The model shows, in effect, that this pattern may result without such strategic behavior.

The results have important implications. James [1990] states that public universities face greater uncertainty with respect to their funding than private universities, because they are dependent for that much of that funding on the whim of a small group of people – state executives and legislators. My model suggests that there are two constituencies affected differently by the relative uncertainty of life at the public university: subsidized students whose cost of attending fluctuates substantially; and subsidizing students, of whom the average academic quality level fluctuates substantially (to the extent that they are "inferior goods"). Public universities may anticipate unique social problems and challenges for their student communities as a result of this mix.

There are other implications for public university administrators. It has been suggested that governing boards should raise nonresident tuition to offset losses of university revenue that follow from decreases in state appropriations. The model in this paper suggests rather that, when nonresident tuition levels already incorporate large margins, the key is to manipulate tuition and admissions standards together to increase nonresident enrollment. Though this may seem undesirable to some public universities, it appears to be the most effective way to avoid substantially raising the tuition or curbing the enrollment of residents.

The predictions of the model should be examined empirically. An econometric analysis that considers common effects across a range of funding shocks would contribute to our understanding of these phenomena. It would also be useful to re-examine the effects of changes in state appropriation levels, taking account of the possibility that different students may be affected by these changes in different ways. Finally, the effect of federal student aid on tuition should be re-examined, accounting for different impacts on different groups of students.

APPENDIX

A brief outline of the proof of each result is provided here. Detailed proofs are available from the author upon request.

Proof of Proposition 1: The university chooses P_i and ϕ_i (i=1,2) to maximize (2) subject to (4). The corresponding Lagrangian function is

$$\begin{aligned} L &= \alpha h_1(\phi_1)\phi_1 Q_1(P_1) + (1-\alpha)h_2(\phi_2)\phi_2 Q_2(P_2) \\ &+ \lambda\left[(c_1 - P_1)\phi_1 Q_1(P_1) + (c_2 - P_2)\phi_2 Q_2(P_2) - F\right] \end{aligned} \tag{A1}$$

where $\lambda < 0$. Assume an interior solution, so that the constraint in (4) is binding. The first-order conditions ("FOCs") are

$$
\begin{aligned}
\tfrac{\partial L}{\partial P_1} &\equiv \alpha h_1 \phi_1 \tfrac{\partial Q_1}{\partial P_1} + \lambda\left[\left(c_1 - P_1\right)\phi_1 \tfrac{\partial Q_1}{\partial P_1} - \phi_1 Q_1\right] = 0 \\
\tfrac{\partial L}{\partial P_2} &\equiv \left(1-\alpha\right) h_2 \phi_2 \tfrac{\partial Q_2}{\partial P_2} + \lambda\left[\left(c_2 - P_2\right)\phi_2 \tfrac{\partial Q_2}{\partial P_2} - \phi_2 Q_2\right] = 0 \\
\tfrac{\partial L}{\partial \phi_1} &\equiv \alpha h_1 Q_1 + \alpha \sigma_1 \phi_1 Q_1 + \lambda\left(c_1 - P_1\right) Q_1 = 0 \\
\tfrac{\partial L}{\partial \phi_2} &\equiv \left(1-\alpha\right) h_2 Q_2 + \left(1-\alpha\right)\sigma_2 \phi_2 Q_2 + \lambda\left(c_2 - P_2\right) Q_2 = 0 \\
\tfrac{\partial L}{\partial \lambda} &\equiv \left(c_1 - P_1\right)\phi_1 Q_1 + \left(c_2 - P_2\right)\phi_2 Q_2 = F
\end{aligned}
\tag{A2}
$$

Totally differentiating the FOCs and applying Cramer's rule, one obtains

$$
\begin{aligned}
\tfrac{\partial P_1^*}{\partial F} &= \tfrac{1}{|H|}\left(\tfrac{\partial^2 L}{\partial P_1 \partial \phi_1}\tfrac{\partial R}{\partial \phi_1} - \tfrac{\partial^2 L}{\partial \phi_1^2}\tfrac{\partial R}{\partial P_1}\right)\left(\tfrac{\partial^2 L}{\partial P_2^2}\tfrac{\partial^2 L}{\partial \phi_2^2} - \left[\tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\right]^2\right) \\
\tfrac{\partial \phi_1^*}{\partial F} &= -\tfrac{1}{|H|}\left(\tfrac{\partial^2 L}{\partial P_1^2}\tfrac{\partial R}{\partial \phi_1} - \tfrac{\partial^2 L}{\partial P_1 \partial \phi_1}\tfrac{\partial R}{\partial P_1}\right)\left(\tfrac{\partial^2 L}{\partial P_2^2}\tfrac{\partial^2 L}{\partial \phi_2^2} - \left[\tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\right]^2\right)
\end{aligned}
\tag{A3}
$$

where $|H|$ is the Hessian of the system, which is constrained to be positive by the second-order condition for a maximum.[18]

The second parenthetical expression is the same in each of equations (A3), while the first parenthetical expression differs between the two. All three expressions may be signed unambiguously, using (1), (3), (A1) and (A2): $\tfrac{\partial^2 L}{\partial P_2^2}\tfrac{\partial^2 L}{\partial \phi_2^2} - \left[\tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\right]^2 > 0$, $\tfrac{\partial^2 L}{\partial P_1 \partial \phi_1}\tfrac{\partial R}{\partial \phi_1} - \tfrac{\partial^2 L}{\partial \phi_1^2}\tfrac{\partial R}{\partial P_1} < 0$, and $\tfrac{\partial^2 L}{\partial P_1^2}\tfrac{\partial R}{\partial \phi_1} - \tfrac{\partial^2 L}{\partial P_1 \partial \phi_1}\tfrac{\partial R}{\partial P_1} > 0$. Therefore, $\tfrac{\partial P_1^*}{\partial F} < 0$ and $\tfrac{\partial \phi_1^*}{\partial F} < 0$. ■

Proof of Proposition 2: From the first equation in the FOCs (A2), and using (1), one obtains $\alpha = \tfrac{-\lambda(c_1 - P_1)}{h_1} - \tfrac{\lambda(a_1 - P_1)}{h_1 \gamma_1}$. $P_1 = c_1$ implies $\alpha = -\tfrac{\lambda(a_1 - c_1)}{h_1 \gamma_1}$. If $\tfrac{\partial P_1^*}{\partial \alpha} < 0$ can be demonstrated, then $\alpha < -\tfrac{\lambda(a_1 - c_1)}{h_1 \gamma_1}$ implies $P_1 > c_1$, proving the claim about relative attractiveness. Using Cramer's rule,

$$
\frac{\partial P_1^*}{\partial \alpha} = \frac{1}{|H|}\left\{
\begin{array}{l}
\left(\tfrac{\partial^2 L}{\partial \phi_1^2}\tfrac{\partial R}{\partial P_1} - \tfrac{\partial^2 L}{\partial P_1 \partial \phi_1}\tfrac{\partial R}{\partial \phi_1}\right)\left\{\left[h_2 + \sigma_2 \phi_2\right] Q_2 \left(\tfrac{\partial^2 L}{\partial P_2^2}\tfrac{\partial R}{\partial \phi_2} - \tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\tfrac{\partial R}{\partial P_2}\right) - h_2 \phi_2 \tfrac{\partial Q_2}{\partial P_2}\left(\tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\tfrac{\partial R}{\partial \phi_2} - \tfrac{\partial^2 L}{\partial \phi_2^2}\tfrac{\partial R}{\partial P_2}\right)\right\} \\
+\left[\left(h_1 + \sigma_1 \phi_1\right) Q_1 \tfrac{\partial^2 L}{\partial P_1 \partial \phi_1} - h_1 \phi_1 \tfrac{\partial Q_1}{\partial P_1}\tfrac{\partial^2 L}{\partial \phi_2^2}\right]\left[\tfrac{\partial R}{\partial P_2}\left(\tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\tfrac{\partial R}{\partial \phi_2} - \tfrac{\partial^2 L}{\partial \phi_2^2}\tfrac{\partial R}{\partial P_2}\right) - \tfrac{\partial R}{\partial \phi_2}\left(\tfrac{\partial^2 L}{\partial P_2^2}\tfrac{\partial R}{\partial \phi_2} - \tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}\tfrac{\partial R}{\partial P_2}\right)\right] \\
+\left[\left(h_1 + \sigma_1 \phi_1\right) Q_1 \tfrac{\partial R}{\partial P_1}\tfrac{\partial R}{\partial \phi_1} - h_1 \phi_1 \tfrac{\partial Q_1}{\partial P_1}\tfrac{\partial R}{\partial \phi_1}^2\right]\left(\tfrac{\partial^2 L}{\partial P_2 \partial \phi_2}^2 - \tfrac{\partial^2 L}{\partial P_2^2}\tfrac{\partial^2 L}{\partial \phi_2^2}\right)
\end{array}
\right\}
$$

Using substitutions from (A2) this reduces to

$$\frac{\partial P_1^*}{\partial \alpha} = \frac{1}{|H|}\frac{\lambda}{(1-\alpha)\alpha}\left(\frac{\partial^2 L}{\partial \phi_1^2}\frac{\partial R}{\partial P_1} - \frac{\partial^2 L}{\partial P_1 \partial \phi_1}\frac{\partial R}{\partial \phi_1}\right)\left[2\frac{\partial^2 L}{\partial P_2 \partial \phi_2}\frac{\partial R}{\partial P_2}\frac{\partial R}{\partial \phi_2} - \frac{\partial^2 L}{\partial \phi_2^2}\frac{\partial R}{\partial P_2}^2 - \frac{\partial^2 L}{\partial P_2^2}\frac{\partial R}{\partial \phi_2}^2\right]$$

The first expression in parentheses is positive, from proof of Proposition 1. Using (1), (3), (4), (A1) and (A2), it can be shown that the expression in brackets is positive, thus, $\frac{\partial P_1^*}{\partial \alpha} < 0$.

Concerning the claim about limited funding, note that $|\lambda|$ sufficiently large and h_1 small lead to $P_1 > c_1$ even for large values of α. $\frac{\partial \phi_1^*}{\partial F} < 0$ implies h_1 gets smaller as funding dwindles. One needs only to show $\frac{\partial \lambda^*}{\partial F} > 0$. Using Cramer's rule, and using the proof of Proposition 1 and symmetry, $\frac{\partial \lambda^*}{\partial F} = \frac{1}{|H|}\left(\frac{\partial^2 L}{\partial P_1^2}\frac{\partial^2 L}{\partial \phi_1^2} - \left[\frac{\partial^2 L}{\partial P_1 \partial \phi_1}\right]^2\right)\left(\frac{\partial^2 L}{\partial P_2^2}\frac{\partial^2 L}{\partial \phi_2^2} - \left[\frac{\partial^2 L}{\partial P_2 \partial \phi_2}\right]^2\right) > 0$. ■

Proof of Proposition 3: (2) may be written $U = \alpha T_1 + (1-\alpha)T_2$, where $T_i \equiv h_i \phi_i Q_i$ is the total quality units from enrolled students in group *i*. If $P_i > c_i$, then $h_i Q_i + \sigma_i \phi_i Q_i = \frac{dT_i}{d\phi_i} < 0$ follows from the FOCs (A2). Thus, the marginal student from group *i* decreases the university's utility. ■

Proof of Proposition 4: Using (5) and (A3),

$$\frac{\partial E_1}{\partial F} = \frac{1}{|H|}\left(\frac{\partial^2 L}{\partial P_2^2}\frac{\partial^2 L}{\partial \phi_2^2} - \frac{\partial^2 L}{\partial P_2 \partial \phi_2}^2\right)\left[\phi_1 \frac{\partial Q_1}{\partial P_1}\left(\frac{\partial^2 L}{\partial P_1 \partial \phi_1}\frac{\partial R}{\partial \phi_1} - \frac{\partial^2 L}{\partial \phi_1^2}\frac{\partial R}{\partial P_1}\right) - Q_1\left(\frac{\partial^2 L}{\partial P_1^2}\frac{\partial R}{\partial \phi_1} - \frac{\partial^2 L}{\partial P_1 \partial \phi_1}\frac{\partial R}{\partial P_1}\right)\right]$$

Given $\frac{\partial^2 L}{\partial P_2^2}\frac{\partial^2 L}{\partial \phi_2^2} - \left[\frac{\partial^2 L}{\partial P_2 \partial \phi_2}\right]^2 > 0$ from the proof of Proposition 1, this takes the sign of the expression in large brackets. Using (1), (3), (A1) and (A2), that expression may be written $Q_1^2 \frac{\partial Q_1}{\partial P_1}\phi_1 \lambda\left[\frac{(\gamma_1+1)(c_1-P_1)+(a_1-P_1)}{\gamma_1}\right]$. Thus, $\frac{\partial E_1}{\partial F}$ takes the sign of $(\gamma_1+1)(c_1-P_1)+(a_1-P_1)$. Using (1), one can write $\varepsilon_1 = \gamma_1 P_1/(a_1 - P_1)$, therefore $\frac{\partial E_1}{\partial F} < 0$ is equivalent to $\frac{P_1 - c_1}{P_1} > \frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$. ■

Proof of Corollary 1: Substituting $\varepsilon_1 = \gamma_1 P_1/(a_1 - P_1)$ and $\frac{P_1 - c_1}{P_1} = \frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$ into the first equation of the

FOCs (A2), and solving for α, one obtains $\alpha = -\frac{\lambda(a_1 - c_1)}{h_1\gamma_1(\gamma_1+2)}$. Because $\frac{\partial P_1^*}{\partial \alpha} < 0$, $\alpha < -\frac{\lambda(a_1 - c_1)}{h_1\gamma_1(\gamma_1+2)}$ implies $\frac{P_1 - c_1}{P_1} > \frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$, which proves the claim about relative attractiveness. Since $|\lambda|$ sufficiently large and h_1 sufficiently small lead to $\frac{P_1 - c_1}{P_1} > \frac{\gamma_1}{\gamma_1+1}\frac{1}{\varepsilon_1}$ even for relatively large values of α, $\frac{\partial \phi_1^*}{\partial F} < 0$ and $\frac{\partial \lambda^*}{\partial F} > 0$ prove the claim about limited funding. ■

Proof of Proposition 5: This follows from Proposition 1 by inspection of (5), given $Q_i > 0$, $\frac{\partial Q_i}{\partial P_i} < 0$, and $\phi_i > 0$. ■

Effect on Net Tuition of an Increase in Demand: The effect of a change in an exogenous market variable, such as demand, may be decomposed in a Slutsky-like manner [Nagler, 2006]. For example, the effect on net tuition of a change in a variable x_1 may be written:

$$\frac{\partial P_i^*}{\partial x_1} = \frac{\partial P_i^D}{\partial x_1} - \frac{\partial R^*}{\partial x_1}\frac{\partial P_i^*}{\partial \bar{R}} \qquad \text{(A4)}$$

The first term on the right-hand side of (A4) is the *substitution effect*, or direct effect of a change in x_1, holding utility constant. The second term is the *income effect*, or indirect effect of a change in x_1 via university resources. The first component in the income effect, $-\frac{\partial R^*}{\partial x_1}$, is the effect of a change in x_1 on the resource constraint, times -1. The second component, $\frac{\partial P_i^*}{\partial \bar{R}}$, is the effect on net tuition of loosening the resource constraint by \$1. Hence, $\frac{\partial P_i^*}{\partial \bar{R}} \equiv \frac{\partial P_i^*}{\partial F}$. It follows that the income effect of a change in exogenous condition x_1 is proportional to the effect of a positive funding shock, and the proportionality factor is $-\frac{\partial R^*}{\partial x_1}$.

Now let us use (A4) to analyze the effect of an isoelastic shift in demand, given by b_i in (1), on net tuition. Given $\partial \varepsilon_i / \partial b_i = 0$ and constant marginal cost, the substitution effect, $\frac{\partial P_i^D}{\partial b_1}$, is zero. This means the effect of a demand shift is solely the income effect, which is proportional to the effect of a positive funding shock. Specifically, $\frac{\partial P_i^*}{\partial b_i} = -\frac{\partial P_i^*}{\partial R}\frac{\partial R^*}{\partial b_i} = -\frac{\partial P_i^*}{\partial F}\frac{\partial R^*}{\partial b_i} = -\left(c_i - P_i\right)\phi_i\left(a_i - P_i\right)^{\gamma_i}\frac{\partial P_i^*}{\partial F}$. This result may be verified directly using comparative static techniques.

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Endnotes

[1] See "Record Donation for Stanford," *The San Jose Mercury News*, accessed on August 6, 2006, at http://www.mercurynews.com/mld/mercurynews/living/education/15178693.htm.

[2] Stanford University's endowment is $15 billion; the donation increases this composite figure by less than 1%. But because professional schools are often treated as autonomous units financially, the 15% figure is probably more relevant.

[3] Though funding inflows often are specifically earmarked – $100 million of Knight's gift, for example, will be devoted to constructing a new campus for the business school – they free up other university funds for general use.

[4] I follow Ehrenberg & Sherman [1984] in making this assumption.

[5] See, e.g., McPherson & Schapiro, 1991; Hauptman & Krop, 1998; Turner, 1998; and Cunningham et al., 2001.

[6] Under certain circumstances, universities may set different full tuition levels to different students, such as resident and non-resident students. For a discussion of tuition differentials, see Balderston [1997].

[7] Unlike Ehrenberg & Sherman [1984], I do not impose the restriction that total quality units never decrease with the number of students admitted, i.e., $\phi h_i'(\phi_i)/h_i > -1$. The assumption is not necessary to ensure positive enrollment in the present model.

[8] See Garvin (1980), Ehrenberg and Sherman (1984), and James (1990).

[9] The role that such preferences play in university decision-making is well-recognized and intuitive. For example, consider a university that favors star athletes in order to raise the institution's profile and to please alumni, or one that favors low-income students to serve altruistic goals of the university's trustees.

[10] It can be shown that all of the paper's results still obtain when a general functional form is used in place of the linear function *h*, except in the case of a highly convex function *h*. Certain expressions that appear in the results are indexed for the degree of convexity when a more general *h* function is used, but their interpretation is preserved. Calculations are available from the author on request.

[11] I assume $c_i < a_i$, that is, that at least some students from group *i* are willing to enroll at a price above the marginal cost of their enrollment.

[12] This proposition and all other model results pertaining to the effects of funding shocks are written in terms of the effect of a *negative* shock (i.e., a *reduction* in funding). The effect of positive funding shock is simply the inverse of what is stated in each case.

[13] See James [1990].

[14] The result parallels Proposition 2.

[15] See, e.g., McPherson & Schapiro [1991], p. 68.

[16] See Appendix.

[17] This appears consistent with Fethke's [2006] result that the gap between nonresident and resident tuition at public universities varies negatively with the (overall) demand for higher education.

[18] Given symmetry, it is sufficient to derive comparative static results for group 1; corresponding conclusions may be drawn for both groups.