Adjusting to Choice

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May 24, 2018

Abstract

The paper develops a model of individual decision-making in which costly cognitive adjustment complements choice of action. Because adjustment is durable, initial positions in consumption ("endowments") matter for future consumption decisions. Thus the theory provides a unified foundation in primitives for the endowment effect and behaviors traditionally attributed to sunk-cost bias and cognitive dissonance; as well as the effectiveness of advertising, and framing effects. Natural experiments that leverage restrictions on the individual's choice set could permit identification and measurement of endowment effects and other adjustment phenomena in the market.

Keywords   bounded rationality; endowment effect; sunk cost fallacy; framing; endogenous preference.

1 Introduction

When we humans act, we tend to adjust to our actions. We buy a home, choose a spouse, or decide to take a position on a political issue. We then – or sometimes in anticipation – get "psyched up," rehearse the best qualities of our selected course, get acclimated to what we are doing, and rationalize.

Adjustment efforts take varying forms. They may be specific and top-of-mind, or broad-based and ambient. A consumer may actively rationalize the additional expense associated with an all-electric vehicle shortly after purchasing a new Tesla. Meanwhile, over several months and almost without being aware of it, the same individual may find he is "growing into" being a Tesla owner, becoming more accustomed to and accepting of the car's various features and thus enjoying them more. Adjustment may occur as an instantaneous and almost imperceptible process, as when the purchaser of a roll-on quickly assembles an argument for choosing deodorant rather than anti-perspirant. Or it may be extended and obvious to all, as in the case of marital engagement. The desire to adjust optimally to an action may motivate people to seek resources external to themselves, such as friends' advice, information on the Internet, or persuasive images in television commercials. Whether or not they seek external inputs to aid adjustment, individuals invest scarce resoures of attention and energy in the process. And while even the smallest purchases engender a modicum of supportive thinking, the bigger the commitment one makes, the harder one endeavors to learn to love it.[1]

There is a substantial amount of experimental evidence on the complementarity of actions and cognitive processes that alter perceptions of actions. Individuals asked to re-rate alternatives following a decision or in anticipation of one increase their ratings of chosen alternatives and in some cases diminish ratings of non-chosen alternatives (Lieberman *et al.* 2001, Kitayama *et al.* 2004, Sharot *et al.* 2010, Wakslak 2012). Studies using functional magnetic resonance imaging (fMRI) indicate changes in preference-related brain activity contemporaneous with the changes in subjects' subjective rating of stimuli accompanying decisions or actions (Sharot *et al.* 2009, Van Veen *et al.* 2009, Izuma *et al.* 2010, Jarcho *et al.* 2011, Qin *et al.* 2011, Kitayama *et al.* 2013, Tompson *et al.* 2016).

Festinger's (1962) theory of cognitive dissonance explains some of these phenomena conceptually in terms of individuals preferring their actions to be aligned with their beliefs; when they are not aligned, the theory contends, people may become uncomfortable and so alter their beliefs to restore a sense of

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[1]Adjustment, as I have defined it, is distinct from search, which has the objective of identifying one's best choice. The purpose of adjustment is to increase utility from a choice one has already determined – or is, in parallel, determining – to be one's best.

comfort. The tri-component model of attitude also reflects the notion that action moves hand-in-hand with adaptive changes in beliefs and feelings (see, e.g., Grimm 2005, Chih *et al.* 2015). This model has been applied extensively to explain consumer behavior and, as such, has formed the basis for a substantial amount of marketing strategy.

Despite the evidence that people adjust to their actions and its general acceptance by psychologists and marketers, economists have yet to incorporate adjustment in a robust way into the individual decision-making model. This likely stems from the fact that adjustment nominally involves a change in tastes. Most economists are reluctant to consider changing tastes because they tend to upend traditional approaches to identifying preferences and measuring welfare. Yet the idea that choices not only reflect, but also create, preferences continues to gain currency in the social sciences (Ariely & Norton 2008, Acharya *et al.* 2016).

In this paper I offer a theory of individual decision-making in which adjustment complements choice. I circumvent some of the thornier issues associated with taste change by modeling a consumer who obtains utility from an adjustment-augmented commodity. In my framework, adjustive thinking quasi-changes preferences in the sense of increasing the consumer's marginal utility for the product; but in the context in which the consumer operates - rationally choosing both a quantity of the product and quantity of adjustment as complementary inputs to utility – tastes may be said to be fixed. An exogenous parameter representing product quality (or, alternatively, consumer-specific taste for the product) affects the marginal utilities of both the product and its adjustment complement. While product consumption only affects utility contemporaneously, adjustive thinking creates a durable stock of product-specific attitude that affects the utility of future consumption.[2]

The key innovation of the paper is to characterize adjustment - hence preference change - as a manifestation of bounded rationality. The individual constructs and rehearses arguments, makes certain facts and perceptions more (or less) salient in consciousness, works on acclimating, and so forth, in order to learn to love an object or activity. Agents in my theory can thus *discretionarily* change their preferences, but to do so requires use of scarce cognitive resources and so is *costly*. Conceiving of preference change in this way affords a simple, portable modeling approach that both accords well with the evidence and allows for cogent consideration of how rational people might trade off efforts to like

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[2]The manner in which thinking in the model creates an enduring stock of adjustment is similar to the mechanism in Becker's (1965, 1985) model in which certain present actions create enduring stocks that are complementary with future actions. Becker's complementary stocks explain diverse behaviors that appear prima facie to represent changing tastes, including addiction and effects stemming from the accumulation of human capital (Stigler & Becker 1977, Becker & Murphy 1988). However, Becker's theory does not admit cognition as a complement to action and so fails to recognize the role played by adjustment in the utility obtained from consumption activities. For further discussion, see section 4.

their various activities better.

People do not only adjust to increase their satisfaction from current and future consumption; sometimes they do so to justify a past decision to themselves. For example, subjects in forced compliance experiments change their beliefs to feel better about actions they took that were incongruent with their beliefs or values at the time (Festinger & Carlsmith 1959). To capture such motivations, I allow regret to play a role in the agent's objective function. To give meaning, in turn, to the notion of regret, I introduce endowments – exogenous actions occurring in the initial period of the model prior to the realization of quality/taste. Endowments can represent choices the individual made under uncertainty or subject to a constraint; they may in the extreme represent actions forced on the individual. Such actions are suboptimal given the realization of quality, or they may appear so in hindsight. The regret-driven individual attempts to use adjustment to directly rationalize the "endowed" decision, in essence rendering it post-hoc optimal.

The model provides a unified foundation in primitives for a number of phenomena that have previously been treated separately. First, it demonstrates that endowments increase consumption of the same good at the margin in future periods – an endowment effect – because they drag adjustment with them. This finding offers an account of the endowment effect more robustly consistent with the evidence than existing explanations (e.g., loss aversion, ownership effects). The effect is unambiguously increased by regret, because the need to justify one's prior "excessive" action quite generally implicates greater (compensatory) adjustment. Second, the model recognizes escalation of commitment as a pattern of repeated action driven by compensatory adjustment. The persistently regretful agent harks back continually to his initial "error," seeking again and again to justify it. He does so on a cognitive level; his rationalization then drives, as its complement, repeated action of the same kind taken initially. The model's predictions in this regard are consistent with experimental evidence that regret fosters repeat purchase behavior by consumers (Mittelstaedt 1969) and more broadly fit with descriptive accounts of escalation of commitment from the literature (e.g., Staw 1976). Third, the model shows that many demonstrations of cognitive dissonance in the experimental literature fit a predictable pattern of compensatory adjustment effects arising from restriction of the individual's choice set for agents who are motivated by regret. This recognition paves the way for new public policy approaches to dissonance-related problems. It also reveals a framework for identifying the broader set of adjustment phenomena - including most notably endowment effects - in the market using natural experiments. Fourth, the model provides a robust rational-agent explanation for a range of marketing practices. In particular it offers an understanding of

why advertising, price promotions, and free introductory offers are observed, have long-lasting effects on demand, and can improve welfare, *even when they do not serve to increase the consumer's information about a product.* An important manifestation of this aspect of the theory is its ability to explain framing effects as *facilitating self-persuasion* in a way that is functionally equivalent to the role of advertising.

In a paper relevant to the present effort, Eyster (2002) analyzes sunk-cost effects using a dynamic decision-making framework in which an agent acts in the present to justify past actions. His model is able to explain scenarios such as Thaler's (1980) classic example of a family that decides to go to a basketball game during a snowstorm, the family noting that they would not have gone had they received the tickets for free rather than purchasing them. Strategic complementarity of the initial action (deciding to purchase the tickets) and a subsequent action that justifies the initial one (attending the game) propels observed sunk-cost "biased" behavior.

The adjustment-to-choice model offers an advance over Eyster's approach in cases where a cognitive layer is essential to understanding the mechanism of post hoc justification. To demonstrate, consider the scenario, described by Akerlof & Dickens (1982), in which individuals face a dissonance-producing decision of whether to work in a hazardous industry and, subsequently, are given the opportunity to purchase safety equipment. Based purely on the complementarity of actions, one would expect the workers to purchase the safety equipment in some cases when the benefits to improved safety do not exceed equipment costs: the adoption of the equipment renders more prudent in retrospect one's decision to work in the industry. Yet, consistent with the anecdotal evidence on safety-related behavior in a range of situations (e.g., motorcycle helmet adoption, hockey headgear use, AIDS testing), workers in such situations typically *avoid* the equipment even when its isolated net benefit is positive. The reason, as described in the discussion on cognitive dissonance in Section 4 of this paper, is that the initial action in such scenarios is inconsistent with the individual's preferences; the resultant regret precipitates compensatory adjustment of those preferences. The adjustment is durable, whence it leads to a reduction in the adoption of future behaviors that might have been rational *but for* the adjustment. Critically, then, it is *cognition* (i.e., adjustment) that, in compensating for the initial problematic action in retrospect, sets in motion the observed behavioral dynamic. The key mathematical result from the model associated with this behavior is that regret reverses the sign of the marginal effect of product quality on adjustment.

In conceiving of an individual who curates his thoughts to make the most of future activities or to minimize regrets from past actions, the adjustment theory naturally relates to the concept of motivated

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reasoning (see Epley & Gilovich 2016 for a survey of the literature). The idea behind motivated reasoning is that certain beliefs are desirable and will be held when it is possible for the individual to hold them rationally. The typical approach is to endogenize cognitions by modeling explicitly the demand for and supply of relevant cognitions.[3] This explicitness can be useful in special decision contexts in which the qualitative characteristics of considered beliefs or actions are relevant to determining the feasibility of certain other actions, as, for example, when a person behaves according to a restrictive moral code so that in the future he will be able to view himself (desirably) as a moral person. In the consumer choice context, however, feasibility as a strategic issue is typically secondary. The relevant question is not whether it is going to be possible to get "psyched up" about a certain action, but *how much* one will, and at what cost. My bounded rationality approach, which posits a reduced-form "cost of cognition,"[4] is well-suited to evaluating the dynamics of action in such bread-and-butter contexts.

The rest of this paper is structured as follows. Section 2 lays out the portable model of individual decision-making involving adjustment to choice. Section 3 introduces endowments and regret. Section 4 applies the model to explain the four core phenomena highlighted above: the endowment effect, escalation of commitment, cognitive dissonance reactions, and marketing practices (cum framing effects). Section 5 briefly takes up empirical testing of the theory. Section 6 concludes by discussing alternative assumptions and some possibilities for future work. The Appendix contains proofs of all results.

2   Consumption and Adjustment

Consider an individual who consumes over an infinite series of periods, indexed t, extending out from an initial period $t = 0$. He must make two decisions each period: how much $x_t \geq 0$ to consume of a good x,[5] and how much "adjustive" thinking $T_t \geq 0$ to engage in in support of that good.[6] Let us initially assume that the thinking decision and the consumption action decision at t are made simultaneously. The good x is not durable: the consumption decision must be renewed each period and the history of

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[3]See, for example, Bénabou & Tirole (2011), p. 806.

[4]The reduced-form adjustment cost is analogous to the reduced-form minimum cost function used in classic profit maximization analysis, which subsumes the input combination decisions needed to produce an output at minimum cost. Similarly a bounded rationality approach to preference change subsumes decisions over cognitions (and potentially behaviors) needed to produce adjustment at minimum cost. The mechanics of these are not essential to predicting dynamic consumption streams, and so need not be made explicit so long as they are convincingly independent of those streams except through their effect on adjustment.

[5]More broadly, good x might be an activity that could be engaged in at varying levels of intensity.

[6]In the concluding section, I discuss the implications of admitting negative thinking ($T_t < 0$) in the model.

previous consumption does not matter directly for current utility. However, thinking creates a durable stock of adjustment, y_t, according to the process

$$
\begin{aligned}
y_t &= (1-\sigma)\, y_{t-1} + T_t,\ t > 0 \\
y_0 &= T_0
\end{aligned}
\tag{1}
$$

in which $\sigma \in (0,1)$. This stock of adjustment is, in turn, complementary with the contemporaneous consumption of x.

Formally, let instantaneous utility $u = u(x,y)$ be a twice continuously differentiable, strictly concave function, whence the complementarity of consumption and adjustment manifests as $u_{xy} > 0$, subject to the further constraint $u_{xx}u_{yy} > u_{xy}^2$. This specification has the intuitive characteristic that the value of adjustment is related to the consumption value attached to the goods in question, so that the first unit of adjustment effort expended on a car would have greater payoff, say, than the first unit of such effort invested in a can of anti-perspirant. For convenience, I will at times express instantaneous consumption utility as the derived function of the quantities of instantaneous activity and thinking, $u(x,T) \equiv u(x,y(T))$.

The consumption problem occurs in the context of a broader economy in which there are many goods and activities in which money and effort can be invested. To focus the analysis, I assume the consumer possesses an invariable, finite supply of effort, $K > 0$, each period. This effort may be allocated to adjust to good x; or to earn labor income, paid in a numeraire commodity, that may be spent on x or on other consumption activities. The price of x, supplied by a competitive industry, is normalized to one. Each unit of numeraire not spent on x garners one unit of utility (via the other consumption activities). I assume that x is just one of many activities of the consumer, whence $x_t, T_t \ll K$ in every t.

At time $t = \tau$, the consumer's preferences are represented by the quasilinear discounted utility function

$$
U(x_\tau, x_{\tau+1}, ...; T_\tau, T_{\tau+1}, ...) =
\tag{2}
$$

$$
\mu u(x_\tau, T_\tau) + K - x_\tau - T_\tau + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau}\left[\mu u(x_t, T_t) + K - x_t - T_t\right]
$$

where $\mu > 0$ is the good or activity's quality level and $\delta \in (0,1)$ is the consumer's discount rate. Given $x_\tau, T_\tau \ll K$, the constraint posed by the finite supply of effort each period does not bind; thus the

consumer's problem is the unconstrained optimization,

$$\max_{T_\tau, x_\tau} (2) \tag{3}$$

Quality is an intrinsic characteristic of the good and does not vary over time. The variable μ might, alternatively, represent the consumer's exogenous taste for, or attitude toward, the good (i.e., in the traditional "fixed" sense), whereby μ is intrinsic jointly to the consumer and the good. This latter interpretation conceives of a consumer whose attitude comprises fixed and discretionary components. The consumer begins in the initial period $t = 0$ with only his fixed component set; whereupon, according to the process in (1), he begins to invest in the discretionary component, adjusting to the activity that he is about to commence.

The consumption process arising out of this complementary utility model may be thought of as similar to driving a car on a cold day. One can turn the car on and drive it productively without warming up the engine, taking the "fixed" components of the car as given and obtaining utility from those. But one might get better results - better contemporaneous acceleration, as well as longer life for the engine - if one invests in warming the car up first. The model allows for two extremes with respect to the ongoing process of adjusting to consumption. With $\sigma = 0$, adjustment does not depreciate at all: once the consumer has finished his initial adjustment to an activity, he is "adjusted," and he does not have to do it again. With $\sigma = 1$, adjustment depreciates completely, whence fresh adjustment is required every period. For the values of σ in between, the setup conceives of a consumption activity that requires some ongoing discretionary cognitive effort in order for the consumer to remain primed, or "psyched up."

In intertemporal models, it is a common feature of the literature for the consumer to be modeled as a sequence of temporal selves who make choices in a dynamic game with one another (e.g., Pollak 1968, Peleg & Yaari 1973, Goldman 1980, and Laibson 1997). This approach lends itself naturally to modeling dynamically inconsistent preferences whereby, for example, the consumer at t would disagree with the tradeoff decision between consumption in $t + 1$ and $t + 2$ that a consumer at $t + 1$ would make. The preferences given by (2) are not dynamically inconsistent; however, I will use a modified objective that does exhibit dynamically inconsistent preferences when I consider regret in section 3. Therefore I adopt here, and carry forward through subsequent analysis, the approach of modeling the infinite-period consumption problem as an infinite game, with an infinite number of players, or "selves," indexed based on their respective periods of control over the consumption and thinking decisions (Laibson 1997). I

seek subgame perfect equilibrium (SPE) strategies of this game. In that context, I will let S_t represent the set of feasible strategies in the game for self t, where a strategy represents a move (x_t, T_t), while $S = \prod_{t=0}^{\infty} S_t$ represents the joint strategy space for all selves.

The following is the main technical result of the core model:

Theorem 1. *The infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve (3) has a unique subgame perfect equilibrium strategy, $s^*(\mu) \in S$, characterized by: (i) a steady-state $(x^*(\mu), y^*(\mu))$ such that $(x_0, T_0) = (x^*(\mu), y^*(\mu))$ and $(x_t, T_t) = (x^*(\mu), \sigma y^*(\mu))$ for all $t > 0$; and (ii) $x_\mu^*, y_\mu^* > 0$.*

Theorem 1 confirms that there is a unique equilibrium path for a discounted-future-utility-maximizing consumer who simultaneously chooses action and complementary adjustive thinking every period starting from the very first period. In a world in which product quality is known from the outset, the unique consumption and adjustment path is a steady state in which both depend positively on the level of quality. The consumer engages in adjustive thinking each period at the intensity level needed to maintain steady-state adjustment given its depreciation rate, σ.

3 Endowments

Now consider a tweak to the setup: suppose x_0 is fixed exogenously at $\bar{x}_0 > 0$ rather than being set by the consumer contemporaneously with adjustive thinking at $t = 0$. This might represent a number of things. The consumer might have committed to a level of consumption before knowing the good's quality level (or his taste for it). Subsequently, quality is revealed, placing him in the position of deciding how much adjustment to do given his prior consumption commitment and the quality level. Or the consumer might be exogenously endowed with a level of consumption, having received the good as a gift or bequest. Or the individual might somehow have been compelled to take an action independent of his tastes or beliefs. In all these cases, the level $\bar{x}_0$ presents itself at the start of the game before anything else is determined - an existing "fact" to which the consumer must adapt his adjustment level.

For the case of endowments a simple corollary to the theorem offers the revised equilibrium existence result:

Corollary 1. *The infinite consumption game in which the consumer at each $t > 0$ chooses (x_t, T_t) to solve (3) and the consumer at $t = 0$ chooses T_0 given x_0 fixed at $\bar{x}_0$ to solve (3) has a unique subgame*

perfect equilibrium strategy, $s^* (\bar{x}_0, \mu) \in S$.

3.1 Equilibrium with Regret

In general, the level of endowed actions will not be consistent with the consumer's realized tastes or the realization of quality and so not on the optimizing steady-state path. That is, $\bar{x}_0 \neq x^* (\mu)$. This gives rise to the possibility that the consumer will feel he erred in having taken the action at its endowed level and will regret his error. He may see himself as responsible and may question his own judgment. Where he is clearly not responsible – having merely acted in a way that was chosen for him by someone else – he may still experience incongruence between his values and his action. In both cases, the experience is unpleasant. The consumer acts to minimize such displeasure.

One may think of the experience of regret as reframing the consumer's problem. A non-regret-driven consumer chooses and adjusts in order to make the most of his present and future consumption opportunities. A regret-driven consumer instead uses his power to choose and adjust in the present in order to minimize the discrepancy between the path he believes he should have chosen in the past and the one he actually chose. It is possible in general that a consumer may embody a little bit of both the forward-looking and the backward-looking individual, whence his decision process takes a hybrid form. The extent to which he acts in a forward-looking or backward-looking way could vary over time, following his moods, pressures imposed by his social environment, or other relevant aspects of his situation.

To fix ideas, let us assume the consumer's objective at $t = \tau$ takes the following general form:

$$\max_{T_\tau, x_\tau | \bar{x}_0} U^R (x_\tau, T_\tau; \beta_\tau) = (1 - \beta_\tau) \left\{ \mu u (x_\tau, T_\tau) + K - x_\tau - T_\tau + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau} \left[\mu u (x_t, T_t) + K - x_t - T_t \right] \right\}$$

$$+ \beta_\tau \left\{ \left[\mu u (\bar{x}_0, T_\tau) + K - \bar{x}_0 - T_\tau \right] - \left[\mu u (x_0^* (T_\tau), T_\tau) + K - x_0^* (T_\tau) - T_\tau \right] \right\} \quad (4)$$

This function weights the quasilinear discounted future utility function in (2) with a second term. That second term in effect revisits the consumption decision from $t = 0$ in hindsight. It is a strictly non-positive loss function that is minimized, at a value of zero, when the endowed level of consumption turns out to have been the optimizing level *given adjustment* at $t = \tau$. The consumer endeavors through this term, in effect, to manipulate the instrument that he currently has available ($t = \tau$ adjustment) to render his earlier decision ($t = 0$ consumption) post-hoc optimal, acting *as if* through current adjustment he could turn back the clock and somehow make better his prior consumption decision.

Here, $\beta_\tau \in [0, 1)$ parameterizes the degree of regret experienced at $t = \tau$. This form anticipates a consumer who not only experiences immediate displeasure (i.e., in $t = 0$) from having erred, but who continues to experience *lingering* displeasure in later periods and to experience a lingering desire to reduce it through adjustment. It allows for maximum generality in specifying how regret lingers over time, allowing the model to represent, for example, a person who feels substantial regret for his decision one period, a lot less the next, and then a subsequent resurgence in later periods. Non-lingering regret is nested as the special case $\beta_\tau = 0 \; \forall \tau > 0$; and other interesting special cases such as a simple decay path (i.e., $\beta_\tau = \beta^\tau$) could also be represented.

I assume the consumer does not experience meta-regret, that is, regret over experiencing regret. For this reason, because the endowment does not affect utility directly except for the current period, the regret component does not include any terms relating to future periods. Moreover, I assume as a baseline that the consumer conjectures naïvely that his future self will not be affected by regret. There are a number of assumptions one could make about future regret conjectures (see, e.g., Eyster 2002), including that the consumer has perfect foresight about the degree of regret he will experience in every future period. I discuss relaxing the naïve consumer assumption in section 6.

The preferences represented by (4) are, in general, dynamically inconsistent. This occurs because the regret-driven part of the consumer cares only about minimizing regret in the current period (say, t) and not about consumption in any future period. This influences the way the overall consumer evaluates the tradeoff between $t + 1$ and $t + 2$. Because the degree of regret generally varies over time, when $t + 1$ arrives, the tradeoff between $t + 1$ and $t + 2$ is viewed differently by the overall consumer.[7] One may check that this characteristic of regret leads to a marginal rate of substitution between periods $t + 1$ and $t + 2$ from the perspective of the decision-maker at t that does not equal the marginal rate of substitution between those same periods from the perspective of the decision-maker at $t + 1$, consistent with the demonstration of dynamic inconsistency proposed by Laibson (1997).

The equilibrium existence result for the regret context depends on complementary restrictions on the convexity of the regret term and the maximal degree of regret. If the regret term is quite convex, then it must be weighted lightly in the objective for the overall objective to be strictly globally concave. If the regret term is not very convex or is concave, then it may be weighted heavily in the objective. Existence is guaranteed by bounding the convexity above, whence one may obtain an upper bound on the degree

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[7]A naïve consumer experiencing any degree of regret over time - even a constant degree - will perceive his preferences to be dynamically inconsistent as he will believe his future self not to be affected by regret.

of regret that guarantees global concavity of the overall objective.

Lemma 1. *Assume $u(x, y)$ such that, for any $\bar{x}_0 > 0$, there exists $Z(\bar{x}_0, \mu) < \infty$ where for all T_τ at all $\tau \geq 0$,*

$$\mu u_{yy}(\bar{x}_0, y_\tau) - \mu u_{yy}(x_0^*(T_\tau), y_\tau) - \mu u_{xy}(x_0^*(T_\tau), y_\tau) x_T^* < Z(\bar{x}_0, \mu)$$

Then it follows there exists $B \in (0, 1)$ such that for any $\beta = (\beta_0, \ldots, \beta_t, \ldots)$ for which $\max_t \beta_t \leq B$, the infinite consumption game in which the consumer at each $t > 0$ chooses (x_t, T_t) to solve (4) and the consumer at $t = 0$ chooses T_0 given x_0 fixed at $\bar{x}_0$ to solve (4) has a unique subgame perfect equilibrium strategy, $s^*(\bar{x}_0, \mu, \beta) \in S$.

3.2 Comparative Statics

With Theorem 1, the associated corollary, and Lemma 1 as technical underpinning, two comparative static results relating to adjustive thinking in the initial period in the presence of an endowment set the stage for applications of the model.

Proposition 1.

1. *For a consumer who does not experience regret in the initial period ($\beta_0 = 0$), adjustive thinking in the initial period increases with quality (complementary adjustment).*

2. *With extreme regret ($\beta_0 \approx 1$), adjustive thinking decreases with quality (compensatory adjustment).*

3. *More generally, $\partial^2 T_0 / \partial \mu \partial \beta_0 < 0$; that is, intensified regret causes adjustive thinking to become increasingly compensatory.*

Proposition 1 establishes that regret reverses the direction of the effect of quality on adjustment in the initial period. The result is intuitive. A higher quality good yields greater benefits to incremental adjustment in consumption utility. Thus, quality "surprises" excite the forward-looking individual, motivating him to invest more in adjusting to the unexpectedly desirable good. I refer to such use of adjustive thinking as *complementary adjustment*. On the other hand, for an individual who is only concerned about minimizing regret, unexpected quality is perceived as a threat. It suggests, viewed in relation to the endowment, that he made a judgmental error and *chose too little* of the good. The regret-afflicted individual is motivated only to reduce that unpleasant perception and to offset the surprise, which he

achieves by adjusting *less* when quality is higher. The offset is achieved in that his reduced adjustment lowers in retrospect the corresponding optimal level of consumption to bring it in line with the consumption quantity he previously chose. I refer to this use of adjustive thinking as *compensatory adjustment*. For cases along the continuum between the extremes of pure regret and pure discounted consumption-utility maximization, a greater weight on regret in the objective implies an increase in the compensatory motive.

The second comparative static result concerns the endowment's effect on initial period adjustment.

Proposition 2. *A larger endowment induces increased adjustive thinking in the initial period (i.e., $\partial T_0 / \partial \bar{x}_0 > 0$). This tendency increases with the degree of initial period regret (i.e., $\partial^2 T_0 / \partial \bar{x}_0 \partial \beta_0 > 0$).*

The more of the good the individual has, the greater the value of adjusting to it. This is true purely from the perspective of consumption utility: if you are going to do more of something, it is all the more important that you love it. Regret implies the same effect: the larger the amount of the endowed good, holding quality constant, the greater the perception that one choose too much of it. This motivates additional compensatory thinking, so that the amount of the good is justified post hoc by the amount of adjustment the individual later committed to it.

To understand why the endowment's effect on adjustive thinking is *greater* the greater one's regret, consider a simple thought experiment. Suppose, to start with, that there is an over-endowment $\bar{x}_0 > x^*$. A regret-free individual will set T_0 higher than what he would choose for x^*. Next, suppose the quality level is increased, ceteris paribus, to the level of quality that would correspond to the endowment if that amount of the good had been chosen with quality known. Based on Proposition 1, the same regret-free individual would set T_0 yet higher. Next, suppose the individual is transformed into one who experiences at least some regret, and simultaneously consider a reduction in the quality level *back* to its previous lower level. Again invoking Proposition 1, one can see that T_0 is either decreased less than it had been previously increased by raising quality, or else increased further still if the degree of regret is great enough. The net observation is that a higher endowment, all else equal, results in greater adjustive thinking the more regretful the individual.

4 Analysis

In the following subsections, I consider several applications of the adjustment-to-choice model.

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4.1 The Endowment Effect

When a person who is forward-looking adjusts to his actions, a remarkable thing happens when he is endowed with a significant amount of a good. He learns to love it. If adjustment is sufficiently durable, these feelings may carry over into future periods, inducing increased consumption of the good in future periods. In particular, the larger the endowment and the more durable the adjustment, the longer the consumption effect extends.

Proposition 3. *(Endowment Effect) Let $(x^*(\mu), y^*(\mu))$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve* (2), *and suppose $\bar{x}_0 > x^*$. Then: (i) the regret-free consumer chooses $y_0 > y^*$; and (ii) if $\bar{x}_0$ large enough, as defined implicitly by*

$$\mu u_y \left(\bar{x}_0, \frac{y^*}{(1-\sigma)^\tau} \right) > \mu u_y (x^*, y^*)$$

then $y_t > y^*$ and $x_t > x^*$ for all $t \leq \tau$.

The endowment effect, under the adjustment model, is a mere possession effect. Merely possessing an object motivates thinking to improve attitude toward the object in hand. Durability of the attitude is all that is needed to motivate persistent behavior going forward. Note that this has nothing to do with the honoring of sunk costs: it follows for a "rational" discounted future utility maximizer from a process by which thinking complements action.

There are two main pre-existing explanations of the endowment effect in the literature. The first, better known in economics, is that individuals have reference-dependent preferences and experience loss aversion relative to their reference point as described by prospect theory (Kahneman *et al.* 1991). That is, the loss of items internally referenced as being in one's possession is registered as more consequential than the gain of the same items not in one's possesion. In this account, reference dependence is taken as primitive, and no attempt is made to explain why parting with something in trade should be viewed as a "loss." The loss aversion account cannot explain ownership effects whereby individuals – whether buyers or sellers, and whether what they are trading is their own or not – value objects more when they own an identical item (Morewedge *et al.* 2009).

The second explanation, developed primarily in the psychological literature, is that individuals value more the things they own simply because they own them (Morewedge *et al.* 2009). In this account, the

increased intrinsic valuation by individuals of items they own is taken as primitive. As such, an ownership account cannot explain why there are no endowment effects associated with money (DellaVigna 2009).

The adjustment approach supplants the primitives in both these accounts and, in doing so, sets up a new explanation more robustly consistent with the evidence. The adjustment model is consistent with observations that experienced traders do not exhibit the endowment effect (DellaVigna 2009): experienced traders are likely to treat non-personally-owned, traded items as mere commodities that they do a business in, whence there is nothing, in their eyes, to be "adjusted to."[8] Meanwhile, money, as a pure medium of exchange, does not bear being adjusted to: there is not a material object with respect to which one may, through thinking, improve one's attitude.

The endowment effect result - that high consumption today induces higher consumption tomorrow - is reminiscent of results that follow from the addiction mechanism identified by Becker & Murphy (1988). There is, however, an important distinction. The scope of Becker & Murphy's "rational addiction" theory is a specific class of goods for which consumption *itself* creates complementarity with future consumption, that is, without any cognitive "leaning in" by the consumer at all. For example, an individual will become addicted to heroin if he uses it because of the inherent qualities of the good - whether he psyches himself up for it or not. The contribution of the adjustment model is to recognize that the broader class of *all* activities can demonstrate some apparent characteristics of addiction, because the boundedly rational individual optimally leans in.

Two additional observations about the model's adjustment-based endowment effect deserve special note. First, because adjustment is durable, the endowment effect has the potential to persist even after the individual no longer owns the item. One may say, in a sense, that ownership *imprints* the consumer. This predicted effect is supported empirically: Strahilevitz & Loewenstein (1998) demonstrated using laboratory experiments that past ownership increases the valuation of objects, with the increase being related to the duration of ownership before loss of the object. As I discuss later in the paper, the adjustment model offers a conceptual basis for measuring this durability characteristic alternatively with a natural experiment. Second, consistent with the numerous interpretations of endowments under the theory, the endowment effect may be expected to rear its head in a variety of contexts. A gift or bequest, forced acquisition, or acquisition of a good before quality is known, all imprint the consumer and increase his valuation of the good, potentially forever. This can have far-reaching implications,

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[8]Kőszegi & Rabin (2006) describe the trader phenomenon in terms of traders' rational expectations that there is a high probability of parting with items they have just acquired. This perspective is fully consistent with the adjustment theory view of a rational adjuster who will not adjust if he does not expect to own the object.

including *inter alia* for the profitability of various marketing strategies, as I discuss in a later subsection.

The endowment effect is unambiguously strengthened by regret, as the following result indicates:

Proposition 4. *(Sweet Lemons) Let $(x^*(\mu), y^*(\mu))$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve (2), suppose $\bar{x}_0 > x^*$, and consider a consumer characterized by non-lingering regret $\beta_\tau = 0 \ \forall \tau > 0$ and $\beta_0 > 0$. Define $\bar{x}_{0\tau}(\beta_0)$ to be the size of endowment needed to induce this consumer to set $x_t > x^*$ for all $t \leq \tau$. Then $\frac{\partial \bar{x}_{0\tau}(\beta_0)}{\partial \beta_0} < 0$.*

Put simply, the greater the degree of the consumer's regret, the smaller the endowment it takes to induce an endowment effect in any given future period. The mechanism, which I call the "sweet lemons" effect, is through the effect of regret in inducing compensatory adjustment. A regretful consumer chooses a product that later turns out not to live up to expectations. He compensates by adjusting *more,* then rationalizes his decision, telling himself in effect that the decision must not have been so bad (i.e., the lemon was "sweet"), because, after all, it motivated him enough to adjust considerably. This behavior leads naturally to ongoing consumption of the "lemon." The more strongly a person feels the need to justify a past action (because he more intensely regrets it), the more he will compensate, creating a larger stock of adjustment that will carry forward further into the future.

The self-justification motive's amplification of the endowment effect proposed in Proposition 4 is an important testable implication of the overall model. If subjects primed with a need to justify their acquisition of an item report or demonstrate a greater valuation of the item, this could provide support for the adjustment account of the endowment effect in preference to the loss aversion account.

4.2   Escalation of Commitment

A range of psychological evidence suggests that real-life decision-makers exhibit "sunk-cost bias," violating the normative principle that rational agents should not take account of sunk costs in making decisions. In particular, individuals who have previously invested more in a course of action may be more likely to continue it, a phenomenon referred to variously as the "Concorde effect" (Dawkins & Carlisle 1976) or "escalation of commitment" (Staw 1976). Recent behavioral economic models have rationalized such behavior when past actions and present actions are strategic complements. This can occur, as discussed in the introduction, as a consequence of agents exhibiting a taste for consistency of action (Eyster 2002); or, say, if previously-sunk costs provide a signal of the value of projects to forgetful

agents who must consider whether to continue investment in the present period (Baliga & Ely 2011).

The adjustment model offers another basis for making sense of escalation-of-commitment-type behaviors, one based more firmly in psychological primitives. They can occur if agents who adjust exhibit a lingering desire to achieve a psychological accommodation of prior errors.

Proposition 5. *(Sweet Lemons Redux) Let $(x^* (\mu), y^* (\mu))$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve (2), suppose $\bar{x}_0 > x^*$, and consider a consumer characterized by lingering regret such that $\beta_\tau > 0$ for some $\tau > 0$. Then $x_\tau > x^*$.*

Decision-makers who feel "invested" (in the sunk-cost sense) in a past course of action are motivated to believe that the prior action was worthwhile. This is ulimately something that is achieved at a cognitive level by rationalization - that is, by compensatory adjustment. The process proceeds along the lines detailed following Proposition 4. Proposition 5 proposes that this process will be invoked in any future period in which there is some regret.

The lingering desire to accommodate prior errors is functional for under-endowments as well, as recognized by the following proposition:

Proposition 6. *(Sour Grapes) Let $(x^* (\mu), y^* (\mu))$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses (x_t, T_t) to solve (2), suppose $\bar{x}_0 < x^*$, and consider a consumer characterized by lingering regret such that $\beta_\tau > 0$ for some $\tau > 0$. Then $x_\tau < x^*$.*

I call this the "sour grapes" proposition because it reflects the situation of the fox in the Aesop parable who sees grapes that are out of his reach. He rationalizes that they are sour and so not worth his while anyway. Suppose the fox is given the chance the next day to eat the grapes. Would he pass them up? Proposition 6 indicates that this depends on whether he experiences a lingering desire to rationalize not having gotten the grapes yesterday, as only then would he continue to perceive them as sour to compensate, and then act upon that assumption.

Note the asymmetry between under-endowments and over-endowments in the model when there is no lingering regret. An over-endowment - that is, an endowment of an amount of the activity greater than what would have been chosen with quality known - causes an endowment effect. An under-endowment can be rectified immediately the following period - given that the consumer knows quality and is free to choose the amount of the activity - whence there is no future reduction in consumption, that is, no

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endowment effect. Lingering regret, however, gets in the way of this correction, causing behavior to be biased downward based on the need to continue to rationalize the downside error from the initial period.

4.3 Cognitive Dissonance

4.3.1 The Model and General Principles

Cognitive dissonance has been defined as what occurs when an individual simultaneously holds two beliefs that are inconsistent (Festinger 1962, Aronson 2004). The term has been used to characterize a range of observed behaviors typically propelled by the individual changing his beliefs in response to being faced with an unsettling decision. For example, in the scenario proposed by Akerlof & Dickens (1982) discussed in the introduction, workers find their decision to work in a hazardous industry unsettling and so adjust their beliefs about the industry's hazards. Subsequently, as a result of their changed beliefs, they choose not to purchase safety equipment. Other economic manifestations of cognitive dissonance referred to in the literature include, among others, failure to set aside optimal amounts for retirement by individuals who prefer not to think about their earning ability being diminished by old age (Akerlof & Dickens 1982); and failure of individuals to adjust effort levels to accomplish a desired task, once it becomes clear that the task will not be as easy as expected (Aronson 2004).

The adjustment model offers an advance in terms of understanding these phenomena and their implications. Cognitive dissonance phenomena generally may be characterized in the model's parlance as involving an endowment, regret accruing to a discrepancy between the endowment and the perceived optimal level of activity, and a corresponding compensatory adjustment reaction. In many experimentally-induced and naturally-occurring cognitive dissonance situations, the endowed action that is discrepant with the individual's initial preferences and so propels the cognitive dissonance reaction is framed in terms of a restriction of the choice set.

To fix ideas, suppose the domain for choice is the unit interval $[0, 1]$ and that quality μ known at the outset is such that the individual's preferred initial action is interior to the interval, to wit, $x_0^* \in (0, 1)$. Define the set $\mathbb{C} \subseteq [0, 1]$ of choice options actually available to the individual. In the case where $\mathbb{C} = [0, 1]$, the individual chooses x_0^* and sets T_0 to optimally adjust to x_0^*. In this situation, he experiences no regret. Put another way, there is no dissonance between the individual's actual action and his preferred action.

Now consider the case of forced compliance, as per the experiments of Festinger & Carlsmith (1959).

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This case may be represented in the model by an individual being compelled to take a greater quantity of action than he would have chosen given its quality level μ. Formally, $\mathbb{C} = [\underline{x}, 1]$ where $1 \geq \underline{x} > x_0^*$. This situation creates the potential for regret: the quantity $\underline{x}$ is "too high" and must be rationalized. To reduce his misgivings, the individual adjusts with greater intensity (as per Proposition 2) to support the endowed action. Given the durability of adjustment, his initial adjustive thinking may decrease misgivings about taking the action in the future, increasing his tendency to engage in it again. Both the hypothesized attitudinal adjustment to the endowed action and increased propensity to take it again are consistent with outcomes observed by Festinger & Carlsmith (pp. 203 and 208).

A variation of the phenomenon is provided by the scenario discussed previously of the workers in the hazardous industry. Suppose a particular worker judges the dangers of the industry a priori as neither extreme nor non-existent, such that he would like to participate in the industry workforce but only tentatively or partially (i.e., $x_0^* \in (0, 1)$). Of course, life does not normally permit a worker to be tentative: you have to take a job, or else pass it up. Here, then, $\mathbb{C} = \{0, 1\}$. If the worker judges that taking the job is his best option of the two available and he takes it, he experiences dissonance: the job is more dangerous than he is comfortable with for a full-time commitment. His response is to engage in adjustive thinking to convince himself the job is not as hazardous as people say; in doing so, he causes his actual action and preferred action to converge. Such beliefs tend to persist, whence the dynamic problem arises. If, in the future, the individual is given the opportunity to purchase safety equipment, the option to do so does not look as attractive as it might have before he started the job.

Note that the occurrence of a cognitive dissonance-related behavioral phenomena hinges on what the adjustment model characterizes as regret, the critical role of which, as established in Proposition 1, is to reverse the direction of the marginal effect of activity quality on engagement in adjustive thinking. That is, it renders the adjustment compensatory. Consider again the worker in the hazardous industry, and suppose that worker does not exhibit regret per the model's parlance. The more hazardous the industry, the less such a worker would engage in adjustive thinking to support it (i.e., because hazards reduce the perceived "quality" of working in the industry, there is less benefit to be had from being "psyched up" to work in it). Meanwhile the regret-burdened worker engages in *more* adjustive thinking the more hazardous the industry is. When safety equipment is later introduced, a non-regret-laden worker sees clearly the low quality of the industry he works in and purchases the equipment; the regret-laden worker does not and therefore fails to see the value of safety equipment.

4.3.2 Public Policy Implications

The adverse effect on social welfare of behaviors that stem from cognitive dissonance is well recognized, particularly in areas relating to public health. Policy analysts understand that attitudes can pose problems when one is trying to get people to take socially or even privately desirable actions. Too often, however, policy approaches take the individual's attitude as a *fait accompli*, whence the logical approach is to compel action despite attitude. Requiring motorcycle riders to wear helmets, workers to purchase safety equipment, and those at risk for disease to get tested or treated - all represent examples of this policy approach.

What may get lost in the conventional policy discussion is the interplay between the choice set and the formation of the attitude. In recognizing that attitudes are naturally formed during the initial decision process, the adjustment theory broadens the scope of available policy tools to include manipulating the choice set (or endowment). One area where such a tool has already received some recognition is in crime interdiction. When one alters the publicized punishment for a crime, one in effect alters the choice set open to the would-be criminal. By proper manipulation of the punishments, one may manipulate attitudes - and indeed *employ* cognitive dissonance - in such a way that crime is reduced.

An illustration is provided by Dickens (1986), who describes an oft-repeated experiment in which children are warned not to play with a desirable toy. One group is threatened with a severe punishment for disobedience, while another is told to expect a mild punishment. Much later, the children are again put in the room with the toy, but this time without the threat of punishment. It is observed that the children previously threatened with the severe punishment are more likely to play with the toy than those threatened with the mild punishment. The standard interpretation of these studies is that those threatened with mild punishment had to justify to themselves their decision not to play with the toy. The adjustment model clarifies the mechanism of this process. The children in the experiment are faced with a binary decision: do not play at all, or play (at all) and be subject to a punishment. (Were a child to "dabble" in playing with the toy, this would not prevent him from being punished, so it is fair to say that "dabbling" is dominated by playing "all out" and does not represent an intermediate option worthy of consideration.) The setup in so doing effectively endows a decision not to play with the toy. A child threatened with a severe punishment does not need an intermediate option: he views playing with the toy as "low quality" activity, whence his decision not to play at all is relatively consistent with his preferences. But in the case of the child threatened with a mild punishment, not playing creates

significant dissonance and requires adjustment. Hence the subsequent changes in the child's behavior after he chooses initially not to play - demonstrating that the manipulation of publicized punishments may deter future "illegal" behavior through its effect on attitudes toward (i.e., preferences for) "crimes."[9] A general prediction might be that punishments set at a level *just* sufficient to deter a target population will be the most likely to deter crime over the long-term and at low cost, because compliance will continue even after penalties are removed or costly enforcement scaled back.

Since choice-set discontinuities can give rise to cognitive dissonance, policy problems arising out of dissonance may sometimes be solved by offering people continuous choices or otherwise reframing the choice set as continuous. One area where this approach might be useful is the problem of attaining a consensus on politically polarized social issues. Take the example of climate change. It is fair to say that the climate change debate is currently framed in binary terms: accept that humans are causing a significant change in the earth's climate, or deny this proposition categorically. In effect, a restricted choice set compels people to "take a side." After choosing which side they are on, individuals then work to reduce their dissonance, adjusting their beliefs to be consistent with their chosen side, and filtering new information as it arrives according to its alignment with their desired mental position.[10] If instead a continuum of positions were tenable, people could select the position most consistent, in their judgment, with the facts known to them.[11] As new information came to light, suggesting that the human role in climate change was either greater or not as great as had previously been believed, individuals would feel free to adjust their position on the issue and their corresponding attitudes. Making such a regime possible would necessitate developing social norms for being receptive to other people's perspectives and encouraging discussion of the issue with greater openness. Adopting new language for speaking about the issue that does not cast choices in binary or otherwise restrictive terms might play a role.[12] At a societal level, progress toward positions aligned with the facts could potentially be greatly facilitated, bringing with it more harmonious relations between people with differing views.

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[9]Another example of the use of the structure of punishments to manipulate criminal behavior via preferences is Gneezy & Rustichini's (2000) famous day-care center intervention, in which they found that imposing a fine *increased* the incidence of parents picking up their children late. The fine changed parents' attitudes toward late pick-up from a stigmatized activity to a transactional one, because they perceived the fine to be a price.

[10]For evidence of individuals behaving in this way, see Sunstein *et al.* (2017)

[11]While people who have been engaged in the debate on climate for years most likely have entrenched positions, issue framing could make a difference in the positions of those (younger) individuals who are new to the issue. Apropos of this, Akerlof & Dickens (1982) note that innovation in a society often originates with outsiders, for whom resistance to changing established beliefs is not an obstacle.

[12]See, for example, Tabuchi, Hiroko, "In America's Heartland, Discussing Climate Change Without Saying 'Climate Change,'" The New York Times, January 28, 2017.

4.4 Marketing Practices and Framing Effects

Traditional economic theories have conceived of two roles for advertising: to provide information about a product, and to persuade consumers to prefer the product. Both propositions have limitations. The information theory cannot explain advertisers' costly efforts devoted to crafting message and image in ads otherwise devoid of informational content.[13] The persuasion theory offers no explanation as to why advertising should elicit a response at all from a rational consumer.

The adjustment theory posits a rational consumer who seeks to increase the utility he obtains from the products he chooses. From this emerges a new primitives-based explanation of persuasive advertising as *facilitating self-persuasion*. One posits advertising in the adjustment framework as an expenditure that reduces consumers' adjustment costs with respect to the advertised product. Intuitively, advertisements provide "fodder" - in the form of helpful arguments, gut appeals, and seductive images - for a consumer who is trying to become as enamored of a product as possible. Through this conception, there is the potential for purely persuasive advertising both to influence demand and to serve an efficient, rather than wasteful, purpose. Such a view breaks with traditional economics, which has perceived only informative advertising as welfare-enhancing.[14]

More formally, consider a slight modification of the model in section 2 in which an advertising expenditure A_t on good x may be agreed upon by the competitive firms in the industry in a given period t, as for example through an industry association.[15] Advertising reduces the cost of adjusting to x, such that the cost of adjustive thinking in t becomes $[1 - \phi(A_t)] T_t$ where $\phi(.)$ defined over $A > 0$ is continuous, increasing and strictly concave with $\lim_{A \to \infty} \phi(A) = \alpha \in (0, 1]$. I continue to assume the firms are price takers (say, because antitrust laws prohibit the industry association from fixing the price), and I further assume an infinitely elastic (horizontal) supply curve such that advertising has no price effect and the price of x remains normalized to one. The consumer's objective at $t = \tau$, where advertising occurs at τ, becomes the following (variant of (2), reducing to (2) at $A_\tau = 0$):

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[13]The theory *has* offered an explanation of non-substantive advertising as providing a signal that the product is of sufficient quality to warrant a costly advertising expenditure (Nelson 1974). But if the purpose is just to show that money is being spent and the ad is not in some measure intended to be persuasive, why should message and image details matter?

[14]Akerlof & Dickens (1982) suggest that advertising may help consumers by furnishing them with an "external justification" for believing that a purchased product meets their needs (p. 317). This idea closely aligns with my notion. For a full analysis of the effects of advertising as self-persuasion in a competitive context, see Nagler (2017).

[15]An example would be milk advertising funded by dairy farmers' associations.

$$U\left(x_{\tau}, x_{\tau+1}, ...; T_{\tau}, T_{\tau+1}, ...\right) =$$

$$\mu u\left(x_{\tau}, T_{\tau}\right) + K - x_{\tau} - \left[1 - \phi\left(A_{\tau}\right)\right] T_{\tau} + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau} \left[\mu u\left(x_t, T_t\right) + K - x_t - T_t\right] \tag{5}$$

I assume the consumer conservatively conjectures $A_t = 0$ for all $t > \tau$, regardless of the level A_{τ}.

From this setup, the basic effects of advertising, summarized in the following remark, follow straightforwardly. In fact, the only necessary assumption about the advertising is that it decreases adjustment costs.

Remark 1. Consider a competitive industry with infinitely-elastic supply and persuasive advertising that decreases adjustment costs. Such advertising increases adjustment to, and consumption of, the good; and whenever such advertising is profitable, it increases welfare.

The dynamic effects of advertising that follow from the setup work in an analogous fashion to the endowment effect: advertising in the model has the ability to influence the future to an extent determined by the size of the initial "event." That is, a one-time advertising expenditure can increase the consumption stream for a product over any duration, so long as advertising's capacity to influence adjustment costs, given by α, is complete enough; and the expenditure is of sufficient size.

Proposition 7. *(Advertising Quasi Endowment Effect) Let $\left(x^*\left(\mu\right), y^*\left(\mu\right)\right)$ be the unique steady-state equilibrium in the infinite consumption game in which the consumer at each $t \geq 0$ chooses $\left(x_t, T_t\right)$ to solve (5) for $A_{\tau} = 0$. Define*

$$\Lambda_{\tau}\left(x^*, y^*; \delta, \sigma\right) \equiv \sum_{i=0}^{\tau-1} \delta^i \left(1 - \sigma\right)^i \left[u_y\left(x^*, y^*\right) - u_y\left(x\left(\frac{y^*}{\left(1-\sigma\right)^{\tau-i}}\right), \frac{y^*}{\left(1-\sigma\right)^{\tau-i}}\right)\right]$$

where $x\left(\frac{y^*}{\left(1-\sigma\right)^{\tau-i}}\right)$ is the optimizing value of x given $y = \frac{y^*}{\left(1-\sigma\right)^{\tau-i}}$. If the initial advertising expenditure A_0 is large enough, as defined implicitly by $\frac{\phi(A_0)}{\mu} > \Lambda_{\tau}$, and provided that $\frac{\alpha}{\mu} > \Lambda_{\tau}$, then $y_t > y^*$ and $x_t > x^*$ for all $t \leq \tau$.

Proposition 7 may be understood technically as follows. Each term in Λ_{τ} is the weighted difference between the marginal utility of adjustment at the steady-state versus its counterfactual level if the adjustment stock were inflated a certain number of periods at its depreciation rate, and consumption were adjusted optimally. One may observe that, as τ is incremented, the component terms in Λ_{τ} grow

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larger, and an additional positive term is added to the sum. Thus $\Lambda_{\tau+1} > \Lambda_\tau$ for any $\tau > 0$. Recognizing this, the progressive sum of terms may be thought of as an increasing hurdle: the advertising expenditure must be large enough, given the quality-adjusted productivity of advertising in reducing adjustment costs, to exceed the hurdle corresponding to the value of τ. Intuitively, the advertising investment A_0 decreases adjustment costs, leading to greater adjustive thinking in $t = 0$; to induce increased consumption in any period τ, it is necessary to advertise enough to induce sufficient $t = 0$ thinking to build an adjustment stock whose depreciated level will remain above y^* out to τ. Proposition 7 in effect establishes that there is a mechanism by which one-shot advertising can create persistent demand, even under the conservative assumption that current advertising does not (positively) influence consumers' conjectures about future advertising.[16]

The adjustment model makes way for a more coherent economic understanding of marketing practices other than advertising as well. The practice of giving free product (e.g., samples, trial-basis services, short-term memberships) has traditionally been viewed in economics as providing an incentive to consumers to learn more about the product.[17] Thus giveaways can be an efficient aid to search in a world of consumers with strictly fixed preferences. But if learning about a product is the only motivation for such practices, then why are further coupons and discounts offered to people who have already redeemed previous offers? Viewed through the lens of adjustment, giveaways play another role: they build an *endowment.* The consumer who receives a free one-month membership to a fitness club rationally adjusts to use of the club facilities; because such adjustment is durable, her marginal utility from use of the club is increased in future periods and he is more likely to see the benefit of purchasing a long-term membership. Coupons, discounts, and price promotions all function in a similar way: they create incentives for the consumer to establish an endowment that will lead, through the endowment effect, to future (full-price) purchases. Marketers have long recognized these sorts of benefits and, in fact, are more prone to talk in terms of dynamic preference-smithing effects on consumers (e.g., creating a habit, building a relationship) than they are to think in terms of simply providing the consumer with more information as a basis for making an optimal decision.

Framing effects, according to which different ways of presenting a choice elicit different choices, embody another aspect of marketing intuition: that characteristics not only of the product but of its context - or more broadly, the *situation* as well as the stimulus - are fundamental to inducing consumers

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[16]The simple framework presented here abstracts from the competitive effects and price effects of adjustment-facilitating advertising. These are worthy of separate investigation. See the concluding section.

[17]See, for example, the discussion in the introduction of Sun (2011).

to respond (Aylesworth & MacKenzie 1998, Meloy 2000). Like advertising, framing may be usefully viewed through the lens of faciliating self-persuasion. Consider the following passage from Rabin (1998, p.37):

> Framing effects can often be viewed as heuristic errors - people are boundedly rational, and the presentation of a choice may draw our attention to different aspects of a problem, leading us to make mistakes in pursuing our true, underlying preferences... But sometimes framing effects cut more deeply to economists' model of choice: More than confusing people in pursuit of stable underlying preferences, the "frames" may in fact partially *determine* a person's preferences.

The adjustment theory rationalizes Rabin's notion that frames determine preferences. As with persuasive advertising messages, elements of the way in which choices are presented may provide helpful fodder when a person is seeking motivation or justification for liking a choice he intends to commit to. This is perhaps easiest to see with the framing of a default option: a consumer might rationalize that such an option is "recommended" (McKenzie *et al.* 2006). The so-called "preference reversals" induced by context effects, whereby adding options to the choice set changes the perception of existing options, may be explained in terms of similar, if more complex, self-persuasion logic. Consider the compromise effect observed by Simonson (1989). In a set of three options, the middle option is chosen more often than when it is paired with a single alternative option, violating the principle that substitution effects are necessarily nonpositive. Here, the framing provided by the three-option set facilitates considerably the decision-maker's adjustment to the middle option: he reasons that that option, which offers an apparent compromise on two desired attributes regarding which a tradeoff is essential, will be the most satisfactory. This reasoning leads to the option being preferred post-adjustment. Thus what appears to be a preference reversal is actually seen to be consistent with stable preferences, once adjustment is taken into account.

5 Empirical Testing

Because adjustment is continuous, is correlated with contemporaneous action, and induces changes in future actions, choice-set restrictions imposed through experimental designs provide a natural opportunity to observe it. If one can restrict the choices of an experimental group, normally unobservable

adjustments may be measured through differences in the future behaviors they are predicted to create, relative to a control group. Indeed, all the examples of experimental demonstrations of cognitive dissonance described in subsection 4.3 involve such designs.

This recognition points to the further possibility of demonstrating adjustment behaviors out in the market using *natural* experiments that involve choice-set restrictions. Using the setup of subsection 4.3, consider again the unrestricted set of possibilities for action $x_0 \in [0,1]$, and now let us suppose the propensity to take the action in question depends monotonely on an observable individual characteristic s, which is a continuous variable.[18] Without loss of generality assume $\frac{\partial x_0^*}{\partial s} > 0$. Suppose that the stock of adjustment y_0, a complement to x_0, is measurable in terms of observable actions by the individual (or, alternatively, by a set of attitudes revealed through answers to questions that one could ask the individual).[19] If the de facto choice set is restricted - say, to $\mathbb{C} = \{0,1\}$ - then one has a natural experiment. Focus on those individuals whose levels of s place them close to the transitional value of x_0^* at which the individual would switch from choosing 0 to 1, given the restriction. If those persons choosing 1 exhibit significantly different values of y_0 from those choosing 0, then one has demonstrated conclusively that the decision concerning x_0 has *caused* the change in attitude and any resultant behaviors. The analysis may be conducted formally using regression discontinuity designs.[20]

Another approach uses a temporal restriction of the choice set. Consider the announcement of a new product feature. A consumer's decision to purchase a product not possessing the innovated feature, if made prior to the announcement of the new feature, may be viewed as an endowment. The model predicts increased adjustment by endowed consumers in support of the unimproved product, leading to an increased propensity going forward to purchase products that do not have the innovated feature. Properly controlling for all other relevant factors, it should be possible to test whether such an endowment effect is present in this case. In fact, one could potentially demonstrate persistence of an endowment effect well beyond the period of ownership of the initial product. Along these lines, one might hypothesize - and also test - that consumers who purchased their first cell phone prior to the introduction of smartphones would, all else equal, be more likely to purchase a "flip phone" going forward. Indeed, intuition and casual observation suggest such patterns of habituated consumer behavior.

Relaxations of choice-set restrictions of various kinds may pose similiar temporal natural experiments

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[18]More generally, of course, s could be a vector of individual characteristics.

[19]Note that the variables used to measure y_0 are distinguished from the characteristic s in that they are endogenous while s must be convincingly exogenous.

[20]For a fuller discussion of the regression discontinuity approach, see, e.g., Angrist & Pischke (2008).

for measuring adjustment. Examples include the entry of a new brand into the market, or a loosening of regulations that opens new options for action by decision-makers.

6 Conclusion

This paper has presented a generally applicable theory of individual decision-making based on adjustment to choice. Rather than reiterate the findings of the model, I will use this section to discuss the effect of certain alternative assumptions and to propose some promising directions for future research focused on applications of the framework.

6.1 Negative Thinking

In the model, analysis is restricted to a domain in which the quantity of good purchased and the quantity of adjustive thinking each period are both nonnegative (i.e., $x_t, T_t \geq 0$). While $x_t \geq 0$ is noncontroversial, it is at least worth entertaining whether to allow thinking to be negative, that is, for the consumer to be able to invest costly cognitive effort to like a good *less*. Clearly, a consumer whose preferences are represented by (2) will never wish to engage in negative thinking. Similarly, a regret-driven consumer would never set $T_t < 0$, so long as at least some minimal amount of (positive) adjustive thinking must be invested to get any utility out of consumption, that is, if $u(x_t, 0) = 0$ for all x_t. To see this, consider in (4) the extreme case in which $\beta_\tau \approx 1$ and $\bar{x}_0 = 0$. Even in the event μ is very large, $T_\tau = 0$ renders the zero endowment optimal, thus reducing regret to zero. Minimal adjustive thinking as a prerequesite for consumption utility makes some intuitive sense, in view of psychological research indicating that the deployment of attention is essential to the experience of pleasure and pain (Kahneman & Sugden 2005, Dolan & Kahneman 2008).

If instead one allows for a consumer who gets utility out of a good without any adjustive thinking at all, negative thinking may be rationalized.[21] Formally, $u(\hat{x}_\tau, 0) = k > 0$ for some $\hat{x}_\tau > 0$ implies that quality level $\hat{\mu} = \frac{1}{k} > 0$ renders $\hat{x}_\tau$ optimal given $T_\tau = 0$ as per the first-order condition $\hat{\mu} u(\hat{x}_\tau, 0) = 1$. It follows that any endowment $\hat{x}_\tau > \bar{x}_0 \geq 0$ leads to regret for any regret-driven consumer; if β_τ is sufficiently close to 1, one can see in (4) that the consumer would strictly prefer some $T_\tau < 0$ to $T_\tau = 0$, assuming T could be reduced below zero at some bounded positive cost. Thus regret-driven consumers

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[21]Consider that even a consumer who does no thinking at all about food still gets sustenance from it, hence an undeniable benefit.

will actually "poo poo" a good to relieve regret if they have "erred" significantly by buying too little of the good given the quality realization.

None of the results presented in this paper are affected when we admit negative thinking. Of particular note, the asymmetry between under- and over-endowments noted near the end of subsection 4.2 still follows when $T_t < 0$ is allowed: regardless of whether $u\left(.\right)$ is convex in T for $T_t < 0$ (the most intuitively sensible assumption) or else concave globally, the consumer will engage in sufficient adjustive thinking to bring the adjustment stock up to $y^*\left(\mu\right)$ and will rectify the under-endowment immediately once there is no lingering regret. (Technically, this follows because the maximum for $\beta_t = 0$ remains unique and is unchanged relative to the case where thinking is constrained to be positive.)

6.2 Perfect Foresight about Future Regret

My results relating to regret-driven consumers were based on the assumption that the consumer naïvely conjectures that he will not experience regret in the future. Suppose instead that the consumer experiences regret in one future period $t = \tau > 0$ and that the $t = 0$ consumer - more specifically, his non-regret-driven self - accurately predicts the extent of his regret in that period. (More generally the consumer might experience - and his $t = 0$ self might accurately ancitipate - regret in multiple future periods. But the results for the single-period future regret case are equivalent and provide a simpler illustration.) It is intuitive that such a consumer would wish to set adjustment to compensate for what he considered the errors of his future regret-driven self. The nature of his strategic adjustment behavior would depend on whether $x_0 < x^*$ or $x_0 > x^*$. In the case of $x_0 < x^*$, the $t = 0$ consumer believes his future regret-driven self would set adjustment too low. Because adjustment is durable, he may compensate fully by choosing y_0 large enough to induce a sufficiently large y_τ for the relevant $t = \tau$. Balancing his relative concerns for utility in the periods $t < \tau$ against period $t = \tau$, he would set current adjustive thinking higher than if he did not anticipate future regret, but in general lower than the amount needed to fully compensate for that regret. In the case of $x_0 > x^*$, the $t = 0$ consumer believes his future regret-driven self would set adjustment too high. However, there is nothing he can do about this. If the $t = 0$ consumer were even to set $y_0 = 0$, his future regret-driven self would set adjustive thinking sufficiently high to induce the level dictated by his regret, regardless. Thus the asymmetry between under- and over-endowments shows up in the fact that a strategic non-regret-driven self is able to manage the principal-agent problem arising with his regret-driven future self in the former case, but not the latter - in essence, because he cannot "push on a string." More broadly, one can see that relaxing my assump-

tion about naïve regret conjectures does not change directionally the results discussed in section 4. The theory, as modified by the alternative assumption, still predicts that regret will lead to a strengthened endowment effect and escalation of commitment.

6.3 Further Work

Allowing future consumption in the adjustment model to depend directly on present consumption decisions could provide new insights into self-control problems.[22] The traditional approach assumes that differences in the degree to which individuals procrastinate or seek immediate gratification flow mainly from differences in general attitudes toward the future, reflected typically in single-parameter or two-parameter (i.e., β, δ) discount rates.[23] But the notion of general time-preference is inconsistent with evidence that self-control problems are specifically treatable: for example, individuals who are put through a regimen of goal visualization are more likely to achieve their goals (Cheema & Bagchi 2011). Such findings suggest an alternative model in which complementary adjustment to future goals in the present increases the valuation of those goals, tipping the balance in favor of present decisions that support achievement of the goals (e.g., saving now, staying in school, spending more time studying). Thus, individuals who find such adjustment easier, and for whom adjustment stocks depreciate more slowly, are the ones more likely to exhibit successful self-control.

The present paper's analysis of adjustment was restricted to a partial equilibrium context involving focus on a single product. Extending the adjustment model's approach to a multi-product context would allow for the analysis of *lifestyle* choice - that is, the simultaneous choice of attitudes towards multiple, potentially related product or activity choices (*ensembles*). A general equilibrium model is the proper structure for considering the broader effects of advertising (i.e., in fostering lifestyles, rather than simply stimulating demand for specific products) and other inter-market effects of relevance.

Equilibrium in differentiated product markets has traditionally been modeled with spatial frameworks such as the seminal "beach" proposed by Hotelling (1929). These models represent product offerings as fixed or endogenous locations along a continuum, and consumers' heterogeneous fixed preferences are similarly represented by location. A given consumer will generally not find an ideal choice among the available products, in that there will not be a product offering at his precise location, whence his utility loss from consuming something less-than-ideal is represented by a "transportation cost" from

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[22]I thank Hanna Halaburda for pointing out this application.

[23]O'Donoghue & Rabin (1999) observe that whether individuals are naïve or sophisticated about their self-control problems can also exert a distinct influence on their behavior.

his location to the location of the nearest product. A model of competition in differentiated products with adjustment to choice could provide for improved predictions regarding the price and market share outcomes accruing to various relevant exogenous factors. Introducing adjustment into the standard spatial framework implies a consumer whose location is not fixed, but who "moves closer" to his chosen product, trading off transportation cost against adjustment cost. Advertising as a facilitator of consumer self-persuasion, along the lines of the subsection 4.4, could be modeled as a straightforward add-on in this context, allowing for its competitive and price effects to be better understood. Nagler (2017) and Nagler (2018) offer exploratory treatments along these lines, but there is more work to be done.

Finally, the empirical measurement of adjustment, along the lines of section 5, could provide a proof of concept for the adjustment theory, particularly as a unifying framework. It could, moreover, expand our ability to recognize instances of the range of phenomena that I have proposed here as manifestations of adjustment. I offer two examples. First, as demonstrations of cognitive dissonance have previously only been made using lab experiments, future research that leverages natural experiments could expand the scope for recognizing manifestations of dissonance. Second, there has been limited experimental evidence showing an endowment effect beyond the period of current ownership.[24] Approaches using natural experiments, such as the temporal restriction example offered in section 5, appear to be a natural fit for showing that endowment effects, consistent with the theory presented in this paper, arise as the broader "have-you-ever-owned-it" effects of a durable cognitive adjustment process. Natural experiments involving new product or feature introductions could demonstrate, in particular, that endowments have the potential to impact real-world markets over a long horizon.

A   Appendix

Proof of Theorem 1.   The optimization problem is the same every period, so fix $t = \tau$ and consider the problem for self τ of solving (3). This player recognizes that his move (x_τ, T_τ) will influence the subsequent moves of future selves. Specifically, his choice of T_τ will influence future decisions because it affects the future stock of adjustment which in turn influences the preferences of future selves; his choice of x_τ has no bearing on future selves' preferences or decisions. A subgame perfect strategy takes account of how the choice of T_τ affects responses in all $t > \tau$ by treating T_t as a function $T_t(T_\tau)$ for all $t > \tau$ when choosing the optimal T_τ.

---

[24]Strahilevitz & Loewenstein (1998), mentioned earlier in the paper, offers one example.

Observe from (2) that self τ's objective function nests the objective function of each $t > \tau$. One may rewrite (2) accordingly as

$$U_\tau \equiv \mu u \left(x_\tau, T_\tau\right) + K - x_\tau - T_\tau + \delta U_{\tau+1} \tag{A.1}$$

whence the nesting of future selves' objectives for $t > \tau + 1$ follows by induction. Observe also that T_t for $t > \tau$ appears in self τ's objective function only within U_t. It follows that one may write

$$\frac{\partial U_\tau}{\partial T_\tau} = \left.\frac{\partial U_\tau}{\partial T_\tau}\right|_D + \sum_{t=1}^{\infty} \frac{\partial U_\tau}{\partial U_{\tau+t}} \left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D \frac{\partial T_{\tau+t}}{\partial T_\tau} \tag{A.2}$$

where the "D" indicates the direct effect, not through the choice of T by another self.

For an interior solution $\left.\frac{\partial U_{\tau+t}}{\partial T_{\tau+t}}\right|_D = 0$ for all $t \geq 1$ in (A.2). Thus by the envelope theorem $\frac{\partial U_\tau}{\partial T_\tau} = \left.\frac{\partial U_\tau}{\partial T_\tau}\right|_D$, that is, the overall effect of T_τ on utility is equal to the direct effect, ignoring effects of T_τ through the $T_{\tau+t}$ that are already being optimized by future selves.

With this in mind, the first-order conditions of (3) may be written

$$\mu u_x = 1 \quad ; \quad \mu u_y = 1 \tag{A.3}$$

As the problem is the same in every period, the first-order conditions yield the same $\left(x^*\left(\mu\right), y^*\left(\mu\right)\right)$ interior solution in all periods $t \geq 0$. It follows that $T_0 = y^*\left(\mu\right)$ and $T_t = \sigma y^*\left(\mu\right)$ for all $t > 0$. Note that the constant marginal cost of adjustment implies the consumer engages in adjustment in each period t until the level $y^*\left(\mu\right)$ is reached, regardless of y_{t-1}. It is moreover easy to see from the first-order conditions that $x_\mu^*, y_\mu^* > 0$.

One now must show that this interior solution is the unique maximum. The derivatives of (A.1) needed to evaluate the Hessian with respect to this objective, reflecting the relevant values of (x, y) at which they are evaluated, are

$$\frac{\partial U_\tau}{\partial x_\tau} = \mu u_x \left(x_\tau, y_\tau\right) - 1 \quad ; \quad \frac{\partial^2 U_\tau}{\partial x_\tau^2} = \mu u_{xx} \left(x_\tau, y_\tau\right)$$

$$\frac{\partial^2 U_\tau}{\partial x_\tau \partial T_\tau} = \mu u_{xy} \left(x_\tau, y_\tau\right) \quad ; \quad \frac{\partial U_\tau}{\partial T_\tau} = \mu u_y \left(x_\tau, y_\tau\right) + \delta \left(1 - \sigma\right) \mu u_y \left(x_{\tau+1}, y_{\tau+1}\right) + \ldots - 1$$

$$\frac{\partial^2 U_\tau}{\partial T_\tau^2} = \mu u_{yy} \left(x_\tau, y_\tau\right) + \delta \left(1 - \sigma\right)^2 \mu u_{yy} \left(x_{\tau+1}, y_{\tau+1}\right) + \ldots \tag{A.4}$$

To simplify notation, I drop the arguments for $t = \tau$ and show only arguments for $t > \tau$. The Hessian is

$$
\begin{aligned}
|H| &= \mu^2 \begin{vmatrix} u_{xx} & u_{xy} \\ u_{xy} & u_{yy} + \delta (1 - \sigma)^2 u_{yy} (x_{\tau+1}, y_{\tau+1}) + \dots \end{vmatrix} \tag{A.5} \\
&= u_{xx} u_{yy} - u_{xy}^2 + u_{xx} \delta (1 - \sigma)^2 u_{yy} (x_{\tau+1}, y_{\tau+1}) + \dots > u_{xx} u_{yy} - u_{xy}^2 > 0
\end{aligned}
$$

which follows from the strict concavity of $u(.)$. Since $\frac{\partial^2 U}{\partial x_\tau^2} < 0$ and $\frac{\partial^2 U}{\partial T_\tau^2} < 0$, $U_\tau(x_\tau, T_\tau)$ is strictly concave. This establishes the solution (x_τ^*, y_τ^*) to the first-order conditions as the unique interior absolute maximum for τ whence, by induction, this is true for all $\tau \geq 0$.

To dispense with the possibility of a corner solution: it was noted previously that the unique interior solution is $(x_t^*, y_t^*) = (x^*(\mu), y^*(\mu))$ for all $t \geq 0$, whence $T_t^* = \sigma y^*(\mu)$ for all $t > 0$. A corner solution of $T_1 = 0$ could result if and only if $(1 - \sigma) y_0^*(\mu) > y^*(\mu)$. But this would imply $y_0^*(\mu) > y^*(\mu)$, which is a contradiction of the unique interior solution for $t = 0$. Thus no corner solution will result in $t = 1$. By similar argument, a corner solution in any $t = \tau$ requires $y_{\tau-1}^*(\mu) > y^*(\mu)$, a contradiction of the interior solution for the prior period. By induction, the interior solution is the only solution for all periods.

This confirms $(x_0, T_0) = (x^*(\mu), y^*(\mu))$ and $(x_t, T_t) = (x^*(\mu), \sigma y^*(\mu))$ for all $t > 0$ constitutes the unique subgame perfect equilibrium strategy for the infinite game.

Proof of Corollary 1. Without varying the setup for Theorem 1 substantively, one may treat self 0 as two selves, one (0_x) who sets x_0 and another (0_T) who subsequently sets T_0, both with μ known. Both receive the same payoff, given by (2) for $\tau = 0$, subsequent to 0_T's move. It is obvious that the theorem holds for this trivial variation. Relative to this, an endowment constitutes an out-of-equilibrium deviation for self 0_x. The subgame that follows this deviation has a unique subgame perfect equilibrium strategy, per Theorem 1, by definition of a subgame perfect equilibrium.

Proof of Lemma 1. The derivatives of (4) needed to evaluate the Hessian with respect to this objective are:

$$\frac{\partial U^R}{\partial x_\tau} = (1 - \beta_\tau)\left(\mu u_x\left(x_\tau, y_\tau\right) - 1\right); \quad \frac{\partial^2 U^R}{\partial x_\tau^2} = (1 - \beta_\tau)\,\mu u_{xx}\left(x_\tau, y_\tau\right)$$

$$\frac{\partial^2 U^R}{\partial x_\tau \partial T_\tau} = (1 - \beta_\tau)\,\mu u_{xy}\left(x_\tau, y_\tau\right)$$

$$\frac{\partial U^R}{\partial T_\tau} = (1 - \beta_\tau)\left(\mu u_y\left(x_\tau, y_\tau\right) - 1 + \delta\left\{\left[(1 - \sigma) + T'_{\tau+1}\left(T_\tau\right)\right]\mu u_y\left(x_{\tau+1}, y_{\tau+1}\right) - T'_{\tau+1}\left(T_\tau\right)\right\}\right.$$

$$\left. + \delta^2\left\{\left[(1 - \sigma)^2 + (1 - \sigma)\,T'_{\tau+1}\left(T_\tau\right) + T'_{\tau+2}\left(T_\tau\right)\right]\mu u_y\left(x_{\tau+2}, y_{\tau+2}\right) - T'_{\tau+2}\left(T_\tau\right)\right\} + \ldots\right)$$

$$+ \beta_\tau\left(\mu u_y\left(\bar{x}_0, y_\tau\right) - \mu u_y\left(x_0^*\left(T_\tau\right), y_\tau\right)\right)$$

$$\frac{\partial^2 U^R}{\partial T_\tau^2} = (1 - \beta_\tau)\left(\mu u_{yy}\left(x_\tau, y_\tau\right) + \delta\left\{\left[(1 - \sigma) + T'_{\tau+1}\right]^2 \mu u_{yy}\left(x_{\tau+1}, y_{\tau+1}\right)\right\}\right.$$

$$\left. + \delta^2\left\{\left[(1 - \sigma)^2 + (1 - \sigma)\,T'_{\tau+1} + T'_{\tau+2}\right]^2 \mu u_{yy}\left(x_{\tau+2}, y_{\tau+2}\right)\right\} + \ldots\right)$$

$$+ \beta_\tau\left[\mu u_{yy}\left(\bar{x}_0, y_\tau\right) - \mu u_{yy}\left(x_0^*\left(T_\tau\right), y_\tau\right) - \mu u_{xy}\left(x_0^*\left(T_\tau\right), y_\tau\right)x_T^*\right] \quad \text{(A.6)}$$

where $x_0^*\left(T_0\right)$ arises implicitly as the solution to $\mu u_x\left(x_0^*, y_0\right) = 1$. (Thus terms $\mu u_x\left(x_0^*\left(T_0\right), y_0\right)x_T^* - x_T^*$ drop out of the parentheses on the bottom line of $\frac{\partial U^R}{\partial T_\tau}$.)

Suppose first that $(1 - \sigma)\,y_{\tau-1} < y_\tau^*$, such that the solution arising from the first-order condition is an interior solution and candidate unique solution. Note, in this case, that all the T_t' terms in (A.6) are constants that either take a value of zero or $-(1 - \sigma)^{t-\tau}$. The value taken by each of these terms follows from the form of the state equation (1) and depends specifically on the size of the base value of T_τ, which determines in turn whether each T_t is a corner or interior solution. Suppose first that T_τ is relatively small; then the depreciated value of adjustment $(1 - \sigma)\,y_\tau$ will be less than the optimizing value of $y_{\tau+1}$, whence there will be an interior solution for $T_{\tau+1}$ such that $T'_{\tau+1}\left(T_\tau\right) = -(1 - \sigma)$. It follows that $T'_{\tau+t}\left(T_\tau\right) = 0$ for all $t > 1$. If, however, T_τ is a bit larger but not too large, $(1 - \sigma)\,y_\tau$ will be large enough to push $T_{\tau+1}$ to a corner solution at zero; T_τ would then be large enough that changes in its value would influence the choice of T in the following period, $\tau + 2$, whence $T'_{\tau+2}\left(T_\tau\right) = -(1 - \sigma)^2$ and $T'_{\tau+t}\left(T_\tau\right) = 0$ for all $t > 2$. And so on. It can be shown therefore that each squared expression in square brackets in $\frac{\partial^2 U^R}{\partial T_\tau^2}$ is nonnegative; more precisely, they take the value $(1 - \sigma)^{2t}$ up to a threshold t and zero thereafter. Overall, the sum of the curly bracketed terms within the non-regret component

of $\frac{\partial^2 U^R}{\partial T_\tau^2}$ (i.e., the portion that is weighted by $1 - \beta_\tau$), which I shall refer to as "NR" for convenience, is unambiguously negative. The regret component (weighted by β_τ), which I shall refer to as "R", is ambiguously signed.

Dropping the arguments for $t = \tau$ to simplify notation, the Hessian may be written

$$|H| = \mu^2 \begin{vmatrix} (1 - \beta_\tau) u_{xx} & (1 - \beta_\tau) u_{xy} \\ (1 - \beta_\tau) u_{xy} & (1 - \beta_\tau) (u_{yy} + NR) + \beta_\tau R \end{vmatrix}$$

$$= \mu^2 (1 - \beta_\tau)^2 \left[u_{xx} u_{yy} - u_{xy}^2 + u_{xx} NR \right] + \mu^2 (1 - \beta_\tau) \beta_\tau u_{xx} R$$

This takes the sign of

$$(1 - \beta_\tau) \left(u_{xx} u_{yy} - u_{xy}^2 + u_{xx} NR \right) + \beta_\tau u_{xx} R \tag{A.7}$$

Given the strict concavity of $u(.)$, $u_{xx} u_{yy} - u_{xy}^2 > 0$, whence it follows that the first term in (A.7) is positive. It is therefore sufficient for signing the entire expression positive, given β_τ sufficiently small, that R be bounded above.

Proof of Proposition 1. Begin by writing the version of (4) for $\tau = 0$:

$$\max_{T_0 | \bar{x}_0} U^R (\bar{x}_0, T_0) = (1 - \beta_0) \left\{ \mu u (\bar{x}_0, T_0) + K - \bar{x}_0 - T_0 + \sum_{t=1}^{\infty} \delta^t \left[\mu u (x_t, T_t) + K - x_t - T_t \right] \right\}$$

$$+ \beta_0 \left\{ [\mu u (\bar{x}_0, T_0) + K - \bar{x}_0 - T_0] - [\mu u (x_0^* (T_0), T_0) + K - x_0^* (T_0) - T_0] \right\} \tag{A.8}$$

Take the first-order condition with respect to T_0, using (A.4):

$$(1 - \beta_0) \left(\mu u_y (\bar{x}_0, y_0) - 1 + \delta \left\{ \left[(1 - \sigma) + T_1' (T_0) \right] \mu u_y (x_1, y_1) - T_1' (T_0) \right\} \right.$$

$$\left. + \delta^2 \left\{ \left[(1 - \sigma)^2 + (1 - \sigma) T_1' (T_0) + T_2' (T_0) \right] \mu u_y (x_2, y_2) - T_2' (T_0) \right\} + \ldots \right)$$

$$+ \beta_0 \left\{ \mu u_y (\bar{x}_0, y_0) - \mu u_y (x_0^* (T_0), y_0) \right\} = 0 \tag{A.9}$$

where, as mentioned in the proof of Lemma 1, $x_0^* (T_0)$ arises implicitly as the solution to $\mu u_x (x_0^*, y_0) = 1$. (Thus terms $\mu u_x (x_0^* (T_0), y_0) x_T^* - x_T^*$ drop out of the last line.) Recall also from the proof of the lemma that all terms $T_\tau' (T_0)$ are constants. Totally differentiating (A.9) and using D_1, D_2, N_{11}, N_{12}, and N_2

to represent respective curly bracketed terms yields

$$
\left[(1 - \beta_0) \left\{ \mu u_{yy} \left(\bar{x}_0, y_0 \right) + \delta \left[(1 - \sigma) + T_1' \left(T_0 \right) \right]^2 \mu u_{yy} \left(x_1, y_1 \right) + \dots \right\} \right.
$$

$$
\left. + \beta_0 \left\{ \mu u_{yy} \left(\bar{x}_0, y_0 \right) - \mu u_{yy} \left(x_0^* \left(T_0 \right), y_0 \right) - \mu u_{xy} \left(x_0^* \left(T_0 \right), y_0 \right) x_T^* \right\} \right] dT_0
$$

$$
= - \left[(1 - \beta_0) \left\{ u_y \left(\bar{x}_0, y_0 \right) + \delta \left[(1 - \sigma) + T_1' \left(T_0 \right) \right] u_y \left(x_1, y_1 \right) + \dots \right\} \right.
$$

$$
\left. + \beta_0 \left\{ u_y \left(\bar{x}_0, y_0 \right) - u_y \left(x_0^* \left(T_0 \right), y_0 \right) \right\} \right] d\mu - \left[(1 - \beta_0) \left\{ \mu u_{xy} \left(\bar{x}_0, y_0 \right) \right\} + \beta_0 \left\{ \mu u_{xy} \left(\bar{x}_0, y_0 \right) \right\} \right] dx_0
$$

$$
\Longleftrightarrow \left[(1 - \beta_0) D_1 + \beta_0 D_2 \right] dT_0 = \left[- (1 - \beta_0) N_{11} - \beta_0 N_{12} \right] d\mu - N_2 dx_0 \quad \text{(A.10)}
$$

This yields, using Cramer's rule, for $\beta_0 = 0$:

$$
\frac{\partial T_0}{\partial \mu} = -\frac{N_{11}}{D_1} = -\frac{u_y \left(\bar{x}_0, y_0 \right) + \delta \left[(1 - \sigma) + T_1' \left(T_0 \right) \right] u_y \left(x_1, y_1 \right) + \dots}{\mu u_{yy} \left(\bar{x}_0, y_0 \right) + \delta \left[(1 - \sigma) + T_1' \left(T_0 \right) \right]^2 \mu u_{yy} \left(x_1, y_1 \right) + \dots} > 0 \quad \text{(A.11)}
$$

For $\beta_0 \approx 1$ - the case of extreme regret in the initial period - y_0^* is set for given $\bar{x}_0$ by implicit solution to $\mu u_x \left(\bar{x}_0, y_0^* \right) = 1$ as the "justifying" value of y_0. This allows one to perform comparative static analysis for this case by totally differentiating $\mu u_x \left(\bar{x}_0, y_0^* \right) = 1$:

$$
\mu u_{xy} \left(\bar{x}_0, y_0^* \right) dT_0 = -u_x \left(\bar{x}_0, y_0^* \right) d\mu - u_{xx} \left(\bar{x}_0, y_0^* \right) d\bar{x}_0 \quad \text{(A.12)}
$$

This yields

$$
\frac{\partial T_0}{\partial \mu} = -\frac{u_x \left(\bar{x}_0, y_0^* \right)}{\mu u_{xy} \left(\bar{x}_0, y_0^* \right)} < 0
$$

To understand what happens as β_0 varies between 0 and 1, one applies Cramer's rule to (A.10) for general β_0 and then differentiates with respect to β_0:

$$
\frac{\partial T_0}{\partial \mu} = -\frac{(1 - \beta_0) N_{11} + \beta_0 N_{12}}{(1 - \beta_0) D_1 + \beta_0 D_2}
$$

$$
\rightarrow \frac{\partial^2 T_0}{\partial \mu \partial \beta_0} = -\frac{[(1-\beta_0) D_1 + \beta_0 D_2](N_{12} - N_{11}) - [(1-\beta_0) N_{11} + \beta_0 N_{12}](D_2 - D_1)}{[(1-\beta_0) D_1 + \beta_0 D_2]^2} \quad \text{(A.13)}
$$

Recognizing that (A.13) takes the sign of the numerator, let us evaluate the numerator. This can be simplified as follows, prior to substitutions:

$$- \left[(1 - \beta_0) D_1 + \beta_0 D_2 \right] (N_{12} - N_{11}) + \left[(1 - \beta_0) N_{11} + \beta_0 N_{12} \right] (D_2 - D_1)$$

$$= - (1 - \beta_0) D_1 N_{12} + (1 - \beta_0) D_1 N_{11} - \beta_0 D_2 N_{12} + \beta_0 D_2 N_{11}$$

$$- (1 - \beta_0) D_1 N_{11} + (1 - \beta_0) D_2 N_{11} - \beta_0 D_1 N_{12} + \beta_0 D_2 N_{12}$$

$$= - (1 - \beta_0) D_1 N_{12} + \beta_0 D_2 N_{11} + (1 - \beta_0) D_2 N_{11} - \beta_0 D_1 N_{12}$$

$$= - D_1 N_{12} + D_2 N_{11}$$

Substitution back yields

$$- \left\{ \mu u_{yy} (\bar{x}_0, y_0) + \delta \left[(1 - \sigma) + T_1^{'} (T_0) \right]^2 \mu u_{yy} (x_1, y_1) + \ldots \right\} \left\{ u_y (\bar{x}_0, y_0) - u_y (x_0^* (T_0), y_0) \right\}$$

$$+ \left\{ \mu u_{yy} (\bar{x}_0, y_0) - \mu u_{yy} (x_0^* (T_0), y_0) - \mu u_{xy} (x_0^* (T_0), y_0) x_T^* \right\} \left\{ u_z z_y (\bar{x}_0, y_0) + \delta \left[(1 - \sigma) + T_1^{'} (T_0) \right] u_z z_y (x_1, y_1) + \ldots \right\}$$

Now evaluate this expression at $\mu = \bar{\mu}$, the value of μ at which the endowment $\bar{x}_0$ would have resulted if $(\bar{x}_0, T_0)$ were chosen simultaenously with μ known, that is, the value of μ for which $x_0^* (T_0) = \bar{x}_0$. This is the appropriate value at which to evaluate $\partial^2 T_0 / \partial \mu \partial \beta_0$, as the expression is then interpreted as showing how regret influences the individual's response to a quality surprise relative to endowment-based expectations. At this value, the second term in curly brackets is zero. This leaves

$$\mu \left\{ u_{yy} (\bar{x}_0, y_0) - u_{yy} (x_0^* (T_0), y_0) - u_{xy} (x_0^* (T_0), y_0) x_T^* \right\} \left\{ u_y (\bar{x}_0, y_0) + \delta \left[(1 - \sigma) + T_1^{'} (T_0) \right] u_y (x_1, y_1) + \ldots \right\}$$

The first and second terms cancel at $x_0^* (T_0) = \bar{x}_0$, leaving

$$- \mu u_{xy} (x_0^* (T_0), y_0) x_T^* \cdot \left\{ u_y (\bar{x}_0, y_0) + \delta \left[(1 - \sigma) + T_1^{'} (T_0) \right] u_y (x_1, y_1) + \ldots \right\} \tag{A.14}$$

An expression for x_T^* comes from totally differentiating $\mu u_x (x_0^*, y_0) = 1$:

$$\mu u_{xx} (x_0^*, y_0) \, dx_0 \quad = \quad - \mu u_{xy} (x_0^*, y_0) \, dT_0$$

$$\rightarrow x_T^* \quad = \quad - \frac{u_{xy} (x_0^*, y_0)}{u_{xx} (x_0^*, y_0)} > 0 \tag{A.15}$$

Thus, one can sign (A.14) as negative. It follows that $\partial^2 T_0 / \partial \mu \partial \beta_0 < 0$.

Proof of Proposition 2. Applying Cramer's rule to (A.10) yields, for $\beta_0 = 0$:

$$\frac{\partial T_0}{\partial \bar{x}_0} = -\frac{N_2}{D_1} = -\frac{\mu u_{xy}\left(\bar{x}_0, y_0\right)}{\mu u_{yy}\left(\bar{x}_0, y_0\right) + \delta\left[(1 - \sigma) + T_1'\left(T_0\right)\right]^2 \cdot \mu u_{yy}\left(x_1, y_1\right) + \dots} > 0$$

For $\beta_0 \approx 1$, one may apply Cramer's rule to (A.12), yielding

$$\frac{\partial T_0}{\partial \bar{x}_0} = -\frac{u_{xx}\left(\bar{x}_0, y_0^*\right)}{\mu u_{xy}\left(\bar{x}_0, y_0^*\right)} > 0$$

To understand what happens as β_0 varies between 0 and 1, apply Cramer's rule to (A.10) for general β_0 and then differentiate with respect to β_0:

$$\frac{\partial T_0}{\partial x_0} = -\frac{N_2}{(1 - \beta_0)D_1 + \beta_0 D_2} \rightarrow \frac{\partial^2 T_0}{\partial x_0 \partial \beta_0} = \frac{N_2(D_2 - D_1)}{[(1-\beta_0)D_1 + \beta_0 D_2]^2} \tag{A.16}$$

The sign of (A.16) takes on the sign of the numerator. Substitution back yields

$$\left\{\mu u_{xy}\left(\bar{x}_0, y_0\right)\right\} \cdot \left\{\mu u_{yy}\left(\bar{x}_0, y_0\right) - \mu u_{yy}\left(x_0^*\left(T_0\right), y_0\right) - \mu u_{xy}\left(x_0^*\left(T_0\right), y_0\right) x_T^*\right.$$

$$\left. -\mu u_{yy}\left(\bar{x}_0, y_0\right) - \delta\left[(1 - \sigma) + T_1'\left(T_0\right)\right]^2 \mu u_{yy}\left(x_1, y_1\right) - \dots\right\}$$

The first bracketed term is clearly positive. The first and fourth terms in the second bracketed expression cancel. Evaluate the second and third, substituting (A.15):

$$-\mu u_{yy}\left(x_0^*\left(T_0\right), y_0\right) - \mu u_{xy}\left(x_0^*\left(T_0\right), y_0\right)\left[-\frac{u_{xy}\left(x_0^*\left(T_0\right), y_0\right)}{u_{xx}\left(x_0^*\left(T_0\right), y_0\right)}\right]$$

$$= -\mu u_{yy}\left(x_0^*\left(T_0\right), y_0\right) + \mu \frac{\left[u_{xy}\left(x_0^*\left(T_0\right), y_0\right)\right]^2}{u_{xx}\left(x_0^*\left(T_0\right), y_0\right)}$$

Multiply through by $-u_{xx}\left(x_0^*\left(T_0\right), y_0\right)$ and the sign is the same. Removing the arguments to save space:

$$-\mu u_{xy}^2 + \mu u_{xx} u_{yy} = \mu\left(u_{xx} u_{yy} - u_{xy}^2\right) > 0$$

Thus $\partial^2 T_0 / \partial \bar{x}_0 \partial \beta_0 > 0$.

Proof of Proposition 3. Let (x^*, y^*) represent the steady-state optimum under the unique subgame perfect equilibrium with no endowment, and let $y_0\left(\bar{x}_0\right) = T_0\left(\bar{x}_0\right)$ represent the level of y_0 arising from

the no-regret first-order condition for T based on $\bar{x}_0$. Observe that $(1 - \sigma) y_0 (\bar{x}_{01}) = y^*$ defines implicitly the threshold level $\bar{x}_{01}$ of the endowment that leads to a corner solution for adjustive thinking in $t = 1$. That is, it is the minimum level that induces the consumer to set $T_1 = 0$: for lower levels of x_0, $y_1 < y^*$ unless $T_1 > 0$. At the threshold itself, the unique interior solution that obtains for $t > 0$ remains unaffected, thus $(x_t, y_t) = (x^*, y^*)$ for $t \geq 1$ even when $(x_0, y_0) = \left(\bar{x}_{01}, \frac{y^*}{1-\sigma} \right)$. Because $y_0 (\bar{x}_{01})$ itself is defined based on the first-order condition for T, use that condition to write

$$\mu u_y \left(\bar{x}_{01}, \frac{y^*}{1 - \sigma} \right) + \delta (1 - \sigma) \mu u_y (x^*, y^*) + \dots \quad = \quad 1 \tag{A.17}$$

(Note that here, because endowments may create corner solutions in the optimization problem of future selves, the full first-order condition for T must be used, rather than the simplified version used in the proof of Theorem 1 based on application of the envelope theorem.) Using the fact that the optimum still obtains at the threshold, one may write,

$$\mu u_y \left(\bar{x}_{01}, \frac{y^*}{1 - \sigma} \right) \quad = \quad \mu u_y (x^*, y^*) \tag{A.18}$$

Now consider that $(1 - \sigma)^2 y_0 (\bar{x}_{02}) = y^*$ defines implicitly the threshold level $\bar{x}_{02}$ of the endowment that leads to a corner solution for adjustive thinking in $t = 2$. Analogous to the above, one needs the first-order condition for T to obtain a precise expression for $\bar{x}_{02}$; however, in that expression it will be necessary to replace (x_1, y_1) with threshold values in place of the optimal values appearing in (A.18). Given the depreciation rate of adjustment, $y_1 = \frac{y^*}{1-\sigma}$. Therefore, because the process is the same every period, there is an expression identical to (A.18) that defines a hypothetical $t = 1$ threshold level of consumption $\bar{x}_{12}$ for adjusting thinking in $t = 2$, whereby $\bar{x}_{12} = \bar{x}_{01}$. Using the first-order condition with respect to T then,

$$\mu u_y \left(\bar{x}_{02}, \frac{y^*}{(1 - \sigma)^2} \right) + \delta (1 - \sigma) \mu u_y \left(\bar{x}_{01}, \frac{y^*}{1 - \sigma} \right) + \dots \quad = \quad 1$$

whereby substituting (A.18) yields

$$\mu u_y \left(\bar{x}_{02}, \frac{y^*}{(1 - \sigma)^2} \right) + \delta (1 - \sigma) \mu u_y (x^*, y^*) + \dots \quad = \quad 1$$

whence, again using the fact that the optimum still obtains at the threshold, one may write

$$\mu u_y \left(\bar{x}_{02}, \frac{y^*}{(1-\sigma)^2} \right) \quad = \quad \mu u_y \left(x^*, y^* \right)$$

By induction, the level of the endowment that corresponds to the threshold of a corner solution for adjustive thinking in $t = \tau$ is defined implicitly by

$$\mu u_y \left(\bar{x}_{0\tau}, \frac{y^*}{(1-\sigma)^\tau} \right) \quad = \quad \mu u_y \left(x^*, y^* \right)$$

For any $\bar{x}_0 > \bar{x}_{0\tau}$, $u_{xy} > 0$ implies

$$\mu u_y \left(\bar{x}_0, \frac{y^*}{(1-\sigma)^\tau} \right) \quad > \quad \mu u_y \left(x^*, y^* \right)$$

and, moreover, $y_t > y^*$ for all $t \leq \tau$; which in turn implies $x_t > x^*$, which is what we sought to show.

Proof of Proposition 4. The proof of Proposition 3 establishes that for the no-regret case there exists a threshold endowment level for every τ that is defined implicitly by $(1-\sigma)^\tau y_0 (\bar{x}_{0\tau}) = y^*$, where $y_0 (\bar{x}_{0\tau}) = T_0 (\bar{x}_{0\tau})$ is in turn defined by the first-order condition with respect to T, evaluated at $\bar{x}_{0\tau}$. Totally differentiating $(1-\sigma)^\tau T_0 (\bar{x}_{0\tau}) = y^*$ yields

$$(1-\sigma)^\tau \frac{\partial T_0}{\partial \bar{x}_{0\tau}} d\bar{x}_{0\tau} \quad = \quad -(1-\sigma)^\tau \frac{\partial T_0 (\bar{x}_{0\tau})}{\partial \beta_0} d\beta_0$$

and, by Cramer's rule,

$$\frac{\partial \bar{x}_{0\tau} (\beta_0)}{\partial \beta_0} \quad = \quad -\frac{\partial T_0(\bar{x}_{0\tau})/\partial \beta_0}{\partial T_0(\bar{x}_{0\tau})/\partial \bar{x}_{0\tau}}$$

The denominator can be signed positive based on Proposition 2. Given $\partial^2 T_0/\partial \bar{x}_0 \partial \beta_0 > 0$ - also from Proposition 2 - the numerator is also positive for $\bar{x}_{0\tau} > x^*$. It follows that $\frac{\partial \bar{x}_{0\tau} (\beta_0)}{\partial \beta_0} < 0$.

Proof of Proposition 5. Consider first the extreme regret case $\beta_\tau = 1$ for some $\tau > 0$. It is clear from analysis of (4) that $\bar{x}_0 > x^*$ implies $y_\tau > y^*$. Now instead suppose $\beta_\tau \in (0,1)$. We consider two cases. Suppose first that $(1-\sigma)^\tau y_0 (\bar{x}_0) > y^*$. Then $y_\tau > y^*$ for $\beta_\tau = 0$, whence by complementarity of x and y, it follows that $x_\tau > x^*$ for $\beta_\tau = 0$. As it has already been established that $y_\tau > y^*$ in

the extreme regret case, then $y_\tau > y^*$ also when $\beta_\tau \in (0,1)$, and we are done. Suppose instead that $(1-\sigma)^\tau y_0(\bar{x}_0) \le y^*$. Then $y_\tau = y^*$ for $\beta_\tau = 0$. But since $y_\tau > y^*$ in the extreme regret case, it follows also that $y_\tau > y^*$ for any $\beta_\tau \in (0,1)$, whence by complementarity of x and y, $x_\tau > x^*$, and we are done.

Proof of Proposition 6. Consider first the extreme regret case $\beta_\tau = 1$. It is clear from analysis of (4) that $\bar{x}_0 < x^*$ implies $y_\tau < y^*$. Now instead suppose $\beta_\tau \in (0,1)$. It is obvious that $(1-\sigma)^\tau y_0(\bar{x}_0) < y^*$. This means $y_\tau = y^*$ for $\beta_\tau = 0$. But since $y_\tau < y^*$ in the extreme regret case, it follows also that $y_\tau < y^*$ for any $\beta_\tau \in (0,1)$, whence by complementarity of x and y, $x_\tau < x^*$, and we are done.

Proof of Remark 1. One may consider the general case of advertising that decreases adjustment costs by treating $\phi(A)$ as merely increasing in A, but not necessarily continuous or concave or observing the assumption concerning the limit as A grows large. Then, using (5), and invoking the envelope theorem as in the proof of Theorem 1, the first-order condition with respect to T for the period τ in which advertising occurs may be written: $\mu u_y = 1 - \phi(A_\tau) < 1$. It follows that adjustment will be greater in $t = \tau$. Given the complementarity of x and y, then, contemporaneous consumption is also increased. The welfare results follow trivially: because the consumer adjusts in equilibrium, and advertising reduces the cost of adjustment, it relaxes the consumer's constraint and so increases utility. The only additional requirement for the welfare effect of advertising to be unambiguously positive overall is that advertising be profitable for firms.

Proof of Proposition 7. Let (x^*, y^*) represent the steady-state optimum under the unique subgame perfect equilibrium with no advertising, and let $(x(\tilde{y}_{01}), \tilde{y}_{01})$ represent the consumption-adjustment pair corresponding to a level of advertising (write as A_{01}) in $t = 0$ just sufficient to induce a corner solution for adjustive thinking in $t = 1$. Using the full first-order condition for T as in the proof of Proposition 3, it follows that

$$\mu u_y(x^*, y^*) + \delta(1-\sigma)\mu u_y(x^*, y^*) + \ldots = 1$$
$$\mu u_y(x(\tilde{y}_{01}), \tilde{y}_{01}) + \delta(1-\sigma)\mu u_y(x^*, y^*) + \ldots = 1 - \phi(A_{01})$$

where the second condition follows because one is dealing with the threshold value of the corner solution. Using the fact that all the terms on the left-hand side in these expressions are equal beyond the first,

one may write

$$
\begin{aligned}
1 - \mu u_y \left(x^*, y^* \right) &= 1 - \phi \left(A_{01} \right) - \mu u_y \left(x \left(\tilde{y}_{01} \right), \tilde{y}_{01} \right) \\
\rightarrow \frac{\phi \left(A_{01} \right)}{\mu} &= u_y \left(x^*, y^* \right) - u_y \left(x \left(\tilde{y}_{01} \right), \tilde{y}_{01} \right) \\
&= u_y \left(x^*, y^* \right) - u_y \left(x \left(\frac{y^*}{1 - \sigma} \right), \frac{y^*}{1 - \sigma} \right)
\end{aligned}
$$

where, for the last line, I again use the fact that $\tilde{y}_{01}$ just induces a corner solution in $t = 1$.

Now consider the level of advertising in $t = 0$, A_{02}, just sufficient to induce a corner solution for adjustive thinking in $t = 2$, and consider the corresponding $\tilde{y}_{02}$. Using the first-order condition for T,

$$
\mu u_y \left(x \left(\tilde{y}_{02} \right), \tilde{y}_{02} \right) + \delta \left(1 - \sigma \right) \mu u_y \left(x \left(\left(1 - \sigma \right) \tilde{y}_{02} \right), \left(1 - \sigma \right) \tilde{y}_{02} \right) + \ldots = 1 - \phi \left(A_{02} \right)
$$

Here, because it reflects the threshold value of the corner solution at $t = 2$, all terms on the left-hand side are equal to those in the steady-state first-order condition beyond the *second*; this means one can write

$$
\begin{aligned}
1 - \mu u_y \left(x^*, y^* \right) - \delta \left(1 - \sigma \right) \mu u_y \left(x^*, y^* \right) &= 1 - \phi \left(A_{02} \right) - \mu u_y \left(x \left(\tilde{y}_{02} \right), \tilde{y}_{02} \right) \\
&\quad - \delta \left(1 - \sigma \right) \mu u_y \left(x \left(\left(1 - \sigma \right) \tilde{y}_{02} \right), \left(1 - \sigma \right) \tilde{y}_{02} \right) \\
\rightarrow \frac{\phi \left(A_{02} \right)}{\mu} &= u_y \left(x^*, y^* \right) - u_y \left(x \left(\frac{y^*}{\left(1 - \sigma \right)^2} \right), \frac{y^*}{\left(1 - \sigma \right)^2} \right) \\
&\quad + \delta \left(1 - \sigma \right) u_y \left(x^*, y^* \right) - \delta \left(1 - \sigma \right) u_y \left(x \left(\frac{y^*}{1 - \sigma} \right), \frac{y^*}{1 - \sigma} \right)
\end{aligned}
$$

By induction,

$$
\frac{\phi \left(A_{0\tau} \right)}{\mu} > \sum_{i=0}^{\tau - 1} \delta^i \left(1 - \sigma \right)^i \left[u_y \left(x^*, y^* \right) - u_y \left(x \left(\frac{y^*}{\left(1 - \sigma \right)^{\tau - i}} \right), \frac{y^*}{\left(1 - \sigma \right)^{\tau - i}} \right) \right] \tag{A.19}
$$

defines the level of advertising $A_{0\tau}$ in $t = 0$ sufficiently large to induce $y_t > y^*$ and $x_t > x^*$ for all $t \le \tau$. Note that existence of such a level of advertising depends on $\frac{\alpha}{\mu}$ being larger than the right-hand side of (A.19); specifically, this is a necessary condition for $\phi \left(A \right)$ being able to get large enough, given its limit, for (A.19) to hold.

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