A COGNITIVE “THUMB ON THE SCALE”: COMPENSATORY THINKING AND ESCALATION OF COMMITMENT

MATTHEW G. NAGLER*  
The City College of New York, mngler@ccny.cuny.edu

This paper presents a theory of sunk-cost bias that accounts for the complementarity of actions and thoughts about actions. Decision-makers choose not just what to do but, through an attention allocation process called elaboration, how much utility they will obtain from what they do. As in earlier models of sunk-cost biases, individuals exhibit limited memory and a preference for consistency – posed here as “error aversion.” “Forward-looking” individuals optimally allocate elaborative efforts while those influenced by error aversion use them to compensate for, and thereby neutralize, past errors. Because decision-makers rely on remembered enjoyment from past choices as a memory kludge for product quality, the elaborative allocation biases arising from error aversion result in escalation of commitment. The results are robust to whether individuals are naïve or sophisticated about their past error aversion and to whether or not they anticipate the influence of elaboration on future consumption choices and therefore behave strategically.

*I am grateful to Heski Bar-Isaac, Michael I. Norton, Daniel Stone, and seminar participants at Hunter College for helpful comments and suggestions. Kelly Page Nelson provided excellent research assistance.
I. INTRODUCTION

If you can’t change your fate, change your attitude. – Amy Tan

_J’ai plus de souvenirs que si j’avais mille ans._ (I have more memories than if I’d lived one thousand years.) – Charles Baudelaire

Traditional economic theory holds that rational agents should not take account of sunk costs in making decisions. Yet a range of psychological evidence suggests that real-life decision-makers violate this normative principle, exhibiting a “sunk-cost bias.” In particular, individuals who have previously invested more in a course of action may be more likely to continue it, a phenomenon referred to variously as the “Concorde effect” (Dawkins and Carlisle 1976) or “escalation of commitment” (Staw 1976). Recent behavioral economic models have rationalized the honoring of sunk costs in two situations: when past actions and present actions are strategic complements and agents exhibit a taste for consistency (Eyster 2002), and when previously-sunk costs provide a signal of the value of projects to forgetful agents who must consider whether to continue investment in the present period (Baliga and Ely 2011).

This paper offers a new theory on the origins of escalation of commitment that incorporates an element absent from previous models: that people think about what they do. To understand the criticality of this element to explaining sunk-cost bias in certain situations, consider the famous “Monty Hall” experiment conducted by Gilovich et al. (1995). Similar to the setup on the classic television game show, “Let’s Make A Deal,” a subject is asked to pick one box out of three for the chance to win a big prize; the other two boxes contain minor “consolation” prizes. After selecting a box, the subject is shown that one of the non-chosen

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1 See Eyster (2002) for a review of this evidence.
boxes contains only a minor prize. The researcher then asks the subject if he would like to switch boxes. A partner of the subject in the decision task, who is actually a confederate of the researcher, steers the subject to the box that contains only a minor prize (a bumper sticker) and not the big prize (a t-shirt).

After learning which prize he had won, the subject is later asked how much money the researcher would have to offer to get him to sell back the bumper sticker. Gilovich and his collaborators found that subjects who arrived at the bumper sticker by switching to it (the “switch” condition) valued it more than those who arrived at it by passing up the opportunity to switch (the “stay” condition). The difference in valuations indicates that something occurred in the “switch” condition that did not occur in the “stay” condition: subjects engaged in dissonance reduction, or what Gilovich and Medvec (1995) call “psychological repair work.” That is, they thought about the prize in a compensatory way, making up for the negative surprise of the prize’s low value by increasing that value, thereby reducing regret concerning the “error” in the decision.

Now consider what would have happened if the subjects were asked later whether they would be willing to buy the bumper sticker for a certain price. Those in the “switch” condition would likely be more willing to take this action than those in the “stay” condition – a demonstration of escalation of commitment and associated sunk-cost bias. And yet clearly there is no strategic complementarity between later purchasing and the previous decision to switch or stay: the complementarity of switch or stay is with the thinking (i.e., dissonance reduction), not the subsequent action. The impetus for the subsequent hypothetical decision to buy another bumper sticker is the remembered valuation of the bumper sticker based on
the outcome of the prior thinking process. The role of thinking in fostering escalation of commitment is supported by other psychological studies (e.g., Middlestaedt 1969).

The paper proposes a theory of consumer coping behavior based on a general model of complementary action and cognition, coupled with the modeling of a dynamic process by which forgetful consumers attempt to recall product quality based on how much they remember enjoying what they previously consumed. The approach reflects a burst of recent evidence indicating that by thinking strategically about what they do (and in some cases supporting their thinking with certain rituals) individuals can influence their level of satisfaction and comfort with the activities engage in. The findings in this literature go beyond the classic evidence of rationalization of past choices observed in studies of cognitive dissonance reduction (e.g., Gilovich et al. 1995):

- Individuals focus their attention in various ways that enhance the utility obtained from consumption, including focusing on key product qualities (Hoegg and Alba 2007, Elder and Krishna 2010); focusing on creating feelings of uncertainty, including uncertainty regarding the duration of an experience (Bar-Anan et al. 2009, Zhao and Tsai 2011); and on categorizing what they are consuming at a greater level of specificity (Redden 2008).
- People engage in rituals, such as “unscrewing” Oreo cookies before eating them, that increase involvement and intrinsic interest in their consumption activities, thereby increasing the utility obtained (Vohs et al 2013). Setting goals relating to a consumption activity may serve a similar purpose (Hsee et al. 2003, Nunes and Dreze 2006).
• If one “feels right” about how a decision was made, one’s utility from the chosen object is increased (Higgins et al. 2003; Avnet and Higgins 2003, 2006), suggesting that selective self-talk concerning the way a consumption decision was reached may be used to increase pleasure from an activity.

• Via a process known as the “IKEA effect,” agents transform their beliefs about products through direct engagement in their production. An individual increases the utility obtained from a product by in effect augmenting it into a “bundle” that incorporates beneficial feelings of competence and confidence that arise through the production ritual. Consequently it is found that individuals value objects they made themselves, all else being equal, more than objects made by others (Norton et al 2011, Mochon et al 2012).

These findings suggest that the traditional “single-process” model of consumer behavior is incomplete. Instead consumers appear to maximize utility through two complementary decision processes: (1) they allocate limited income among consumption activities (e.g., goods and services), as in the traditional model; and (2) they allocate attention across opportunities for constructive thinking – or elaboration – to maximize the utility they will get out of what they consume.3

2 An extensive literature considers the effect of attention limitations (i.e., bounded rationality) on decision-making outcomes. (See, e.g., Nagler 1993, Gabaix and Laibson 2006, Gabaix 2014; additionally, DellaVigna 2009 provides a recent survey of field evidence.) Indeed, very recent work has called attention to attention scarcity as collateral with income scarcity and as being the source of decision errors with respect to income and other allocations (Mullainathan and Shafir 2013). Whereas this literature in effect focuses on the allocation of limited attention across decisions, the present paper examines the allocation of attention across consumption activities per se, whereby individuals determine how much utility they will get out of what they choose.

3 More broadly, as reflected in recent findings on “conceptual consumption” (Ariely and Norton 2009), utility may be thought of as being generated by a complementary mix of physical consumption and conceptual activities whence the physical and conceptual exhibit a highly symmetric relationship to one another. While cognition supports (i.e., elaborates) action, action correspondingly supports cognition. Thus an individual may
We model these complementary processes using a commodity production approach, whereby the consumer combines quantities of physical goods (or, more generally, consumption activities) with corresponding quantities of complementary elaboration to produce “commodities” that generate utility. Consumers must commit to quantities of physical goods under a condition of ex ante uncertainty over the marginal utility contributed by their consumption alternatives (e.g., product quality). Elaborative allocations are made ex post (i.e., relative to revelations about product quality). In a later period, consumers must again make choices among physical goods options, but are forgetful regarding what they previously learned about product quality; they recall only the utility they received from consuming in the previous period.

We consider outcomes for consumers who exhibit error aversion – a distaste for having allocated their income suboptimally across goods – and for consumers that do not. In the context of this framework, the sunk-cost bias mechanisms of Eyster (2002) and Baliga and Ely (2011) appear in modified form. We find that non-error-averse consumers engage in complementary elaboration, elaborating goods more that offer greater marginal utility, all else equal. Such consumers do not think in terms of consumption “errors” when revelations about marginal utility differ from expectations. Meanwhile, error averse consumers engage in compensatory elaboration: the more error averse they are, the more they allocate elaborative

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4 On the commodity production approach to consumer behavior, see Becker (1965, 1985). Along similar lines, and more closely related to our paper, Koszegi (2010) offers a model in which the anticipation of future actions and the actions themselves interactively influence utility. His is a “single-process” choice-of-action model in which individuals knowingly foster anticipation through action choices; thus it differs from our “dual-process” modeling of coordinated action and cognition choices.

5 There is substantial psychological evidence of differences between evaluated (remembered) and experienced (momentary) enjoyment (Fredrickson and Kahneman 1993, Redelmeier and Kahneman 1996). Memory-based evaluations are sometimes referred to as “decision utility,” as they typically form the basis for the decisions people make rather than momentary well-being (Wirtz et al. 2003). For an extended discussion, see Helliwell (2008).
effort at the margin to activities which they have allocated “too much” of to their
consumption basket, while reducing elaboration for activities that were under-allocated.
Thus, through elaboration, they attempt to make amends for their prior consumption
“errors.” This is the cognitive analog of the compensatory behavior exhibited by Eyster’s
(2002) consumers who have a taste for consistency.

The link from current elaboration to future behavior occurs through consumers’
reliance on remembered enjoyment; that is, they rationally interpret the utility they received
from consumption in the previous period as a signal of product quality. It is shown that non-
error-averse consumers receive an unbiased signal and so accurately predict product quality,
choosing quantities optimally in the second period. However, for error averse consumers,
prior compensatory elaboration acts as a “thumb on the scale,” biasing perceptions of product
quality upward for negative surprises and downward for positive surprises. Consequently, the
error averse consumer escalates commitment to bad choices. This outcome is the cognitive
analog of the escalating behavior exhibited by Baliga and Ely’s (2011) consumers who use
sunk costs as a memory kludge.

The next section introduces the model. Section III presents key results based on the
model’s equilibrium and lays out their welfare implications. Section IV considers extending
the model to the case of a strategic agent. Section V concludes.

II. THE MODEL

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6 Evidence of compensatory elaboration is provided by Klaaren et al. (1994), who observed that travelers
reinterpret their vacation memories in ways that are consistent with their original expectations.
7 The potential for remembered enjoyment to bias decision-making has been noted by Kahneman and Riis
(2005).
Consider the consumption process summarized in Figure 1. There are two physical goods in the market, \( X_1 \) and \( X_2 \), priced exogenously at \( p_1 \) and \( p_2 \), respectively. An individual employs cognitive effort to elaborate his consumption of the physical goods (e.g., by focusing attention in a satisfaction-enhancing way on their consumption, focusing on the good aspects of how he came to choose them, etc.). The utility he will receive from his elaborated consumption activities is \textit{ex-ante} uncertain – this may be, for example, because he is unsure initially about his preferences or about product quality. Let us assume for the purposes of exposition it is product quality. Let product quality for each good be described by the stochastic parameter \( \phi_i \sim f_i(\phi) \) on \([0,1]\). The individual makes an inference quality and then decides how much of each physical good he’ll consume. He simultaneously decides how much of his limited effort to devote to generating wages; this decision sets the budget for his consumption of the goods, to wit,

\[
p_1X_1 + p_2X_2 = I = \alpha_wE_w \quad \text{for some } \alpha_w > 0
\]

The choice is subject to the limitation \( E_w \leq E \), where \( E \) is the individual’s total endowment of effort per period. Subsequent to these decisions, true quality becomes known. The individual now decides on the amount of cognitive effort he will devote to elaborating his consumption of each good. This choice \( (E_1, E_2) \) is made subject to the constraint imposed by his residual effort,

\[
E_1 + E_2 + E_w = E
\]

Finally, the individual obtains utility from consuming the \textit{elaborated} goods, \( Z_1 \) and \( Z_2 \).

There are two periods, \( t = 1,2 \), in which the individual chooses goods and elaborates them; in each of these he engages in the series of actions illustrated in Figure 1. In period 1,
the individual is new to the products and so his quality inference for each is based only on the
knowledge of the distribution of the quality parameters. We assume the individual infers

\[ \hat{\phi}_i = \bar{\phi}_i \equiv \int_{0}^{1} \phi_i f(\phi_i) d\phi_i \]  

Period 2 differs from period 1 in that the individual has consumed the goods before. This
enables him to use information about his previous consumption experience to infer \( \phi_i \). The
manner in which he does this is described later.

II.A. Preferences

Let us assume as a baseline an individual who is not error averse and who obtains
utility from consumption and elaboration as described by

\[ U_{NE} = U(Z_1, Z_2; \phi_1, \phi_2) \]  

where the \( Z_i \) are determined by

\[ Z_i = Z_i(X_i, E_i) \]  

The intuition, following Michael and Becker (1973), is to think of (5) as a production
function by which the individual combines quantities of two inputs, physical goods and
elaborative effort, to produce “commodities” that generate utility. A key assumption is that
goods and elaboration need not be combined in fixed proportions, to wit,

ASSUMPTION 1. (Non-fixed proportions, non-satiation). \( \frac{\partial Z_i}{\partial X_i} > 0 \) everywhere, and

\( \frac{\partial Z_i}{\partial E_i} > 0 \) \( \forall X_i > 0 \).
Thus goods may contribute to utility without being elaborated, and elaboration contributes to utility so long as there is at least something to elaborate. Consistent with a typical production process, the inputs exhibit diminishing returns, so that \( \frac{\partial^2 Z_i}{\partial E_i \partial X_i} < 0 \) and \( \frac{\partial^2 Z_i}{\partial E_i^2} < 0 \), and complementarity, so that \( \frac{\partial^2 Z_i}{\partial E_i \partial X_i} > 0 \).

Let us allow commodities 1 and 2, without restriction, to be complements, substitutes, or independent – that is, \( \frac{\partial \mu_i}{\partial Z_{-i}} > 0 \), \( \frac{\partial \mu_i}{\partial Z_{-i}} < 0 \), or \( \frac{\partial \mu_i}{\partial Z_{-i}} = 0 \), respectively, where \( \mu_i \equiv \frac{\partial U}{\partial Z_i} \). \( U \) is assumed to exhibit the properties associated with a well-behaved concave utility function, namely \( \mu_i > 0 \) and \( \frac{\partial \mu_i}{\partial Z_i} < 0 \). The good-specific quality parameter \( \phi_i \) is, by design, a marginal utility shifter for commodity \( i \), and not for \( -i \). Thus, we let \( \frac{\partial \mu_i}{\partial \phi_i} > 0 \) and \( \frac{\partial \mu_i}{\partial \phi_{-i}} = 0 \). The lifting of \textit{ex ante} uncertainty regarding quality, then, leads to the individual recognizing that his marginal utility from each of the commodities is higher or lower than the levels he previously expected, depending on the value of each \( \phi_i \). This variance from expectation is cognized by the individual as an “error” in predicting his preferences; thus if \( \phi_i > \hat{\phi}_i \), such that if the marginal utility of \( i \) is higher than expected, all else equal, the individual infers that he has chosen too little of \( i \).

Another essential assumption is that goods and elaborative effort not only complement each other in producing utility-generating commodities, but in generating utility itself, to wit:

\[
\text{ASSUMPTION 2.} \quad \frac{\partial^2 U}{\partial E_i \partial X_i} > 0 \quad \text{or, equivalently,} \quad \eta_{E_i}^{\mu_i} > |\eta_{E_i}^{\mu_{-i}}|. \quad \text{8}
\]

\[\text{8 See appendix for derivation.}\]
This expression states that the elasticity of \( \frac{\partial Z_i}{\partial X_i} \) with respect to elaborative effort is large in size relative to the elasticity of \( \mu_i \) with respect to elaborative effort. Intuitively, while increments of elaborative effort are complementary with the physical good in production of the utility-bearing commodity, they also increase the quantity of the commodity, thereby diminishing its marginal utility; in this sense they rival the physical good. For the impact of effort on the marginal utility of the good to be positive, it is therefore necessary that the rate at which increments of elaborative effort enhance the contribution of the physical good to the commodity be large relative to the rate at which increments of elaborative effort cause diminishing returns to the commodity to set in in utility.\(^9\)

We assert moreover that the effect of elaborative effort on the marginal utility of the good in which the effort is invested serves as an upper bound on the effect that the effort has on the other good’s marginal utility, accounting for price, that is,

\[
\text{ASSUMPTION 3. } \frac{\partial^2 U}{\partial E_i \partial X_i} \frac{p_i}{p_{-i}} > \frac{\partial^2 U}{\partial E_i \partial X_i} \frac{\eta_{X_i}}{\eta_{X_i}} > \frac{\partial^2 U}{\partial E_i \partial X_i} \frac{\eta_{X_i}}{\eta_{X_i}}.
\]

This assumption simply states that goods are at most imperfect substitutes for each other, and that the imperfection of substitutability carries over into the complementary effect of elaborative effort on the marginal utility of the goods.

As an alternative to the baseline preference structure, consider the following structure in which the individual experiences error aversion:

\[
(6) \quad U_E = U_{NE} - \omega L(S)
\]

\(^9\) It may be noted correspondingly that \( \eta_{X_i} > \left| \eta_{X_i} \right| \) as well.
where

\[ S \equiv \frac{\mu_1 \frac{\partial \phi_1}{\partial x_1}}{p_1} - \frac{\mu_2 \frac{\partial \phi_2}{\partial x_2}}{p_2} \]  

and where \( \omega \in [0,1) \) and parameterizes the intensity of error aversion. It should be evident that \( S \) represents the departure from equality of the marginal utility per dollar of expenditure for the two unelaborated goods, thus error aversion implies disutility from having made an optimal allocation in the goods decision, prior to elaborating. It follows that \( \omega > 0 \) implies a utility function that is backward-looking, in that it positively weights past errors (in light of the realizations of the \( \phi_i \)) with respect to the sunk decision about goods into the subsequent decision in which elaborative effort is to be allocated. The individual is presumed to care, independent of the utility received from consumption, about reducing his disutility accruing to those past errors. We assume an optimal allocation of goods, \( S=0 \), entails no disutility, hence \( L(0) = 0 \).

The key assumption in this context has to do with the characteristics of the loss function \( L \):

ASSUMPTION 4. (Smoothness and convexity). \( L''(S) > 0 \) on \( \mathbb{R} \), while \( L'(S) \geq 0 \) for \( S \geq 0 \) and \( L'(S) \leq 0 \) for \( S \leq 0 \). Moreover, \( L \) is presumed smooth on \( \mathbb{R} \), whence \( L'(0) = 0 \).

II.B. Inferences from previous consumption

As noted above, the decision process in \( t = 2 \) differs from that in \( t = 1 \) in that the individual benefits from being able to infer something about the \( \phi_i \) from his previous
consumption experience. The critical component at this step is that the individual recalls the past imperfectly, not remembering product quality – neither its realization nor the mean of its prior distribution – nor the amount of elaborative effort he devoted to each product. Both quality and elaborative effort are intangible and so are assumed to leave an unclear record. He recalls only the physical quantities of the goods consumed, which are presumably available from tangible records (such as receipts); and the marginal utility of each commodity consumed, because past enjoyment of the commodity (that is, the good as elaborated) is presumed to be salient in memory. Knowing these, the individual draws an inference $\hat{\phi}_i$ about product quality for $i$ based on a supposition as to what his effort investment must have been.

ASSUMPTION 5. In $t = 2$, the individual recalls $(X_1, X_2)_{t=1}$ and $(\mu_1, \mu_2)_{t=1}$, but neither $(E_1, E_2)_{t=1}$ nor $(\phi_1, \phi_2)$. He therefore forms $\hat{\phi}_i[(X_1, X_2)_{t=1} ; (\mu_1, \mu_2)_{t=1} ; \hat{E}_1, \hat{E}_2]$ for each $i$, where $(\hat{E}_1, \hat{E}_2)$ are the individual’s presumptions regarding his effort investments in $t = 1$.

Since the $t = 2$ individual infers $\hat{\phi}_i$ for each $i$ based on $X_i$, $\mu_i$, and presumptions $\hat{E}_i$ about his effort investments in $t = 1$, there is the possibility that the $t = 1$ self might recognize this and set $E_i$ and $X_i$ strategically to influence $\hat{\phi}_i$. Evidence from the literature suggests, however, that people ignore the role their actions have in establishing their preferences (e.g., Strack et al. 1988, Epley and Gilovich 2001, Ariely et al. 2003).
ASSUMPTION 6. (Nash conjecture). The individual in $t = 1$ conjectures that the $t = 2$ self’s inference about preference intensity is invariant to his (i.e., $t = 1$) choices of product and elaborative effort.

In Section IV, we shall consider the effect of relaxing this assumption and allowing for a strategic $t = 1$ agent.

III. EQUILIBRIUM AND WELFARE

The two-period decision problem described in Section II is one of imperfect recall.\textsuperscript{10} Given this, the decision problem would in general be modeled as a signaling game between a first-period self and a second-period self in which we seek sequential equilibria. Given Assumption 6, however, one may solve for the equilibrium in the two periods sequentially: the $t = 1$ agent makes his choice independent of the need to consider the decision of the $t = 2$ agent; and the $t = 2$ agent simply reacts to the $t = 1$ agent’s choices, knowing that his reaction has no impact on what those choices would have been. We will begin by having the $t = 2$ agent assume the $t = 1$ agent is non-error-averse in all cases, but then allow him to conjecture error aversion in the case where the $t = 1$ is error averse. The results of this analysis, as we will show, then pave the way for us to generalize to a situation where the $t = 1$ agent behaves strategically, recognizing the effect of his actions on the $t = 2$ agent’s actions. This last piece will be taken up in Section IV.

\textsuperscript{10} The discussion here owes a significant debt to Baliga and Ely (2011, p. 42). See also Bodner and Prelec (2003).
III.A. Choice of elaborative effort without error aversion

Solution of the model for \( t = 1 \) is by backward induction, starting with the elaboration choices. An individual who is not affected by error aversion makes his elaboration choices to maximize (4) subject to (2), \( \phi_1, \phi_2 \), and to the values of \( X_1, X_2, \) and \( E_w \) set earlier in the period. Recognizing this is how he will behave, the individual earlier in the period takes the resultant \( E_i(\phi_1,\phi_2,X_1,X_2,E_w) \) and chooses physical consumption quantities of the two goods and an amount of effort to devote to income generation through work to maximize expected utility based on (4), subject to (1) and the density function \( f_i(\phi_i) \).

Our main interest is in the elaboration decision; in particular, it is on the effects that the preference parameters \( \phi_1, \phi_2 \) have on the corresponding equilibrium choices of elaborative effort \( E_1, E_2 \). First-order conditions of the constrained maximization with respect to the elaboration decision are

\[
\begin{align*}
\mu_1 \frac{\partial Z_1}{\partial E_1} + \lambda &= 0; \\
\mu_2 \frac{\partial Z_2}{\partial E_2} + \lambda &= 0; \\
E_1 + E_2 &= E - E_w
\end{align*}
\]

Comparative static analysis based on (8) allows us to sign the effects \( \frac{\partial E_i}{\partial \phi_i} \). These, combined with evaluation of the second-order conditions, lead to the following result.\(^{11}\)

**PROPOSITION 1** (Complementary elaboration). Assuming a non-error-averse utility function, for two commodities 1 and 2 (1) a unique optimal allocation of elaborative effort \( (E_1^*, E_2^*) \) exists; and (2) elaborative effort is *complementary*, that is, higher levels of \( \phi_i \) result unambiguously in higher levels of elaborative effort, \( E_i \).

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\(^{11}\) Proofs of all results, where not incorporated into the text, are provided in the appendix.
Elaborative thinking is more productive when product quality is higher: intuitively, it pays more to get psyched up about something that is really good than something that is mediocre. Thus positive product quality surprises lead to greater elaboration while negative product quality surprises lead to less elaboration. Note that this effect has nothing to do with the non-error-averse agent caring about, or taking any account of, having been surprised; he simply reacts to the revealed reality about quality by adjusting his elaboration optimally.

III.B. Choice of elaborative effort with error aversion

As in the non-error-averse case, solution of the model for $t = 1$ is by backward induction, starting with the elaboration choices. The individual affected by error aversion makes these to maximize (6) subject to (7), (2), $\phi_1$, $\phi_2$, and to the values of $X_1$, $X_2$, and $E_W$ set earlier in the period. Anticipating this behavior, the individual earlier in the period takes the resultant $E_i(\phi_1, \phi_2, X_1, X_2, E_W)$ and chooses physical consumption quantities of the two goods and an amount of effort to devote to income generation through work to maximize expected utility based on (6), subject to (1) and the density function $f_i(\phi_i)$.

It is straightforward to observe that the first-order conditions for the goods decision yield $S = 0$ when $\phi_i$ is known. Thus we are able to validate our interpretation of the loss function in (6) as corresponding to departures from the optimal goods allocation.

Again, our main interest is in the elaboration decision. The first-order conditions with respect to that decision are
\[
\begin{align*}
\mu_1 \frac{\partial Z_1}{\partial E_1} - \omega L'(S) \frac{\partial S}{\partial E_1} + \lambda &= 0 \\
\mu_2 \frac{\partial Z_2}{\partial E_2} - \omega L'(S) \frac{\partial S}{\partial E_2} + \lambda &= 0 \\
E_1 + E_2 &= E - E_w
\end{align*}
\] 

Using comparative statics, one may sign the effects of error aversion, \( \omega \), on the \( \frac{\partial E_i}{\partial \phi} \).

These, combined with the evaluation of the second-order conditions, allow us to state the following proposition:

PROPOSITION 2 (Compensatory elaboration). Assuming a utility function with error aversion as per (6), there exists a neighborhood \( N(\phi_1, \phi_2) \in \mathbb{R}^2 \) of \( \left( \bar{\phi}_1, \bar{\phi}_2 \right) \) such that, on \( N(\phi_1, \phi_2) \), (1) a locally optimal allocation of elaborative effort \( (E_1^*, E_2^*) \) exists; and (2) \( \frac{\partial^2 E_i}{\partial \phi \partial \omega} < 0 \), that is, elaborative effort is *compensatory*: as the intensity of error aversion grows, elaborative effort is less responsive to variations in quality relative to expectation.

Unlike the non-error-averse agent, who does not think in terms of hedonic surprises per se, the error averse agent’s elaboration is clearly oriented around aversion to hedonic surprises. When quality of both products is exactly as expected, then \( S = 0 \), which causes the error aversion term to drop out of (6). In this baseline case, elaboration becomes purely complementary. Relative to this baseline, surprise variations in product quality levels lead to *less*-than-complementary variation in elaboration intensity. The departure from the complementary levels occurs as the agent attempts to compensate for his “error” in the goods allocation decision. For example, when product quality exceeds expectations, the error averse agent sees himself as having “bought too little” of the good. To reduce his perception of
error, he elaborates the good *less* than if he were not affected by error aversion. This reduces his marginal utility from the good (relative to the baseline), bringing the marginal utilities contributed per dollar of the goods closer to each other, that is, bringing $S$ closer to zero. If instead product quality falls short of expectations, the error averse agent sees himself as having “bought too much” of the good. To reduce his disutility from this perception, he elaborates the good more than if he were not error averse, increasing his marginal utility from the good (relative to the baseline) and, again, bringing $S$ closer to zero.

**III.C. Effect of elaborative effort choice on future activity choices ($t=2$)**

**III.C.1. Non-error-averse agent**

We turn to considering outcomes in period 2. Consider first the assumption that the $t = 2$ agent presumes his first-period self was non-error-averse. Given this, Assumption 5 provides for a process by which the individual “reverse engineers” his elaborative effort choices and, through them, his product preferences as revealed the previous period.¹²

Formally, the individual observes

\[
\begin{align*}
\mu_1 & \left( Z_1 \left( X_1, E_1^* \left( X_1, X_2, \phi_1, \phi_2 \right) \right), Z_2 \left( X_2, E_2^* \left( X_1, X_2, \phi_1, \phi_2 \right) \right); \phi_1 \right) \\
\mu_2 & \left( Z_1 \left( X_1, E_1^* \left( X_1, X_2, \phi_1, \phi_2 \right) \right), Z_2 \left( X_2, E_2^* \left( X_1, X_2, \phi_1, \phi_2 \right) \right); \phi_2 \right)
\end{align*}
\]

where, $(E_1^*, E_2^*)$ are assumed to have been generated by a non-error-averse decision process, that is, they are the values of the $E_i$ that, given $(\phi_1, \phi_2)$ and $(X_1, X_2)$, maximized (4).

¹² To simplify the notation in what follows, we drop the subscript “$t = 1$” where it would have appeared (i.e., characterizing the individual’s prior period choices of $X_i$ and $E_i$ and the corresponding realizations $\mu_i$).
From Proposition 1, it follows that $E_i^*$ is a monotonic function of $\phi_i$ such that there is a unique value of $E_i^*$ corresponding to each $\phi_i$ given other parameters. Moreover, by assumption, $\mu_i$ is a monotonic function of $\phi_i$. Thus there exist implicit functions $\hat{\phi}_1(X_1, X_2, \mu_1)$ and $\hat{\phi}_2(X_1, X_2, \mu_2)$ representing the individual’s inference of the actual $(\phi_1, \phi_2)$ pair given $(X_1, X_2)$ and $(\mu_1, \mu_2)$.

Intuitively, the $t = 2$ agent observes marginal utilities, and asks himself what levels of quality could have given rise to these. As the agent recalls also the quantities consumed, any variation in marginal utilities must have been a direct result of variation in unobserved elaboration and unobserved quality. Given Proposition 1, there is only one level of elaboration that could have corresponded to the true quality level for each good; this implies quality levels can be unambiguously inferred from the marginal utilities, given the $t = 2$ agent’s conjectures. If the individual is indeed unaffected by error aversion, then these conjectures about the $\phi_i$ are correct, and consequent activity choices are optimal:

**PROPOSITION 3.** Assume the $t=1$ consumer is not affected by error aversion and that $t=2$ consumer assumes this. Then the consumer’s conjectures about product quality are $\hat{\phi}_i = \phi_i$, and future actions $(X_1, X_2)_{t=2}$ based on these estimates are optimal.

**III.C.2. Error averse agent; naïve $t = 2$ conjecture**

Now consider conjectures and actions in the case in which the individual is actually error averse ($\omega > 0$). We assume first that the $t = 2$ agent conjectures (naively) his $t = 1$ self was non-error-averse; thus he infers the quality levels that would correspond to the
elaboration of a non-error-averse individual. This means biases are introduced generally into the conjectures about $\phi_i$. These biases affect activity choices for the $t=2$ agent who, other than his naïveté regarding the $t=1$ agent’s error aversion, is fully rational.

**PROPOSITION 4** (Escalation of commitment I).

(a) In the neighborhood $N\left(\phi_1,\phi_2\right)\in \mathbb{R}^2$ on which Proposition 2 holds, error aversion leads to estimates of $\phi_i$ that are biased downward for $\phi_i > \bar{\phi}_i$ and biased upward for $\phi_i < \bar{\phi}_i$. The magnitude of the bias increases with the intensity of error aversion, $\omega$.

(b) Subsequent consumption is biased downward relative to the optimum for positive surprises $\phi_i > \bar{\phi}_i$ and upward for negative surprises $\phi_i < \bar{\phi}_i$. Consumption biases increase with the intensity of error aversion, $\omega$.

**III.C.3. Error averse agent; sophisticated $t=2$ conjecture**

Now consider the $t=2$ agent who presumes correctly that his $t=1$ self was error averse ($\omega > 0$).\(^{13}\) Now a given observed level of marginal utility for one of the goods, in light of the known quantities consumed, could correspond to *multiple* possible quality levels and corresponding elaboration levels. For example, one possibility is obviously that the $t=1$ self correctly conjectured about quality – some level $\tilde{\phi}$ - and then elaborated at a level that was entirely complementary (i.e., *not* compensatory) – a corresponding level $\tilde{E}$. Another possibility is that true quality, $\bar{\phi}$, was much higher than the value he conjectured, some $\bar{\phi}$, where $\bar{\phi} < \tilde{\phi} < \bar{\phi}$. The error averse $t=1$ agent chooses an elaboration level $\tilde{E}$ that is less

\(^{13}\) For this discussion, we omit subscripts corresponding to goods to maintain simplicity of notation.
than a non-error-averse agent would given $\tilde{\phi}$. It is easy to see that a level $\tilde{\phi}$ generally exists such that the pair $(\tilde{\phi}, \tilde{E})$ is observationally equivalent to $(\tilde{\phi}, \bar{E})$, given observed marginal utility and quantity. Indeed, one could construct a continuum of such observationally equivalent pairs corresponding to the same values of observed variables $(\mu, X)$.

This clearly presents a problem for agent when trying to infer $(\phi_1, \phi_2)$. One can see, however, that each of the possible pairs $(\phi, E)$ corresponds to a certain posterior probability of occurrence, given the prior distribution of $\phi$ that must have given rise to it. The posterior distribution is in fact a mirror image of the prior distribution, but with a variance that depends upon $\omega$. (In the special case $\omega = 0$, that variance is zero, and the inference problem reduces to the deterministic “reverse engineering” discussed in the previous sections.)

Suppose that the $t=1$ agent had chosen, in fact, too large a quantity for one of the goods, given the prior distribution. Since he was error averse, he also elaborated it too much. This elaboration imparts a positive bias to the posterior distribution deduced by the $t=2$ agent: the distribution of values of $\phi$ is in fact lower than the agent deduces, because he has assumed the elaboration that gave rise to the distribution is on average at its lower, complementary level. Given the posterior distribution, then, the $t=2$ agent’s optimization results again in his choosing too high a quantity. The reverse occurs if they agent in $t=1$ had chosen too small a quantity. This gives rise to

**PROPOSITION 5 (Escalation of commitment II).** A $t=2$ consumer who is aware that he exhibited error aversion in $t=1$ and optimizes based on this supposition chooses
consumption levels that have the same directional bias as the levels chosen in \( t = 1 \). The size of the bias is increasing in \( \omega \).

### III.D. Welfare

The characterization of the various effects of error aversion on welfare is straightforward given the preceding analysis. Under the non-error-averse regime, the individual chooses \( E_i \) to maximize (4) given the realizations of the \( \phi_i \). Given the effects of error aversion on the adjustment of \( E_i \) - that is, \( \frac{\partial^2 E_i}{\partial \phi_i \partial \omega} < 0 \) - it follows that

\[
U(Z_1, Z_2; \phi_1, \phi_2)_{\omega = 0} \geq U(Z_1, Z_2; \phi_1, \phi_2)_{\omega > 0} \ \forall (\phi_1, \phi_2),
\]

with strict equality holding only for \( (\phi_1, \phi_2) = (\bar{\phi}_1, \bar{\phi}_2) \). This of course follows from the assumption that the agent’s objective function contains an additional term when he experiences regret, that is, he maximizes (6) rather than (4). Thus in equilibrium in the error aversion case, whenever there is a surprise to \( \phi_1 \) or \( \phi_2 \), the individual experiences a loss in hedonic utility due to the suboptimal (from a hedonic perspective) allocation of elaborative effort.

Second, rearranging the first-order conditions in (9), one obtains

\[
\mu_1 \frac{\partial Z_1}{\partial E_1} - \omega L'(S) \frac{\partial S}{\partial E_1} + \omega L'(S) \frac{\partial S}{\partial E_2} = \mu_2 \frac{\partial Z_2}{\partial E_2}
\]

whereas the corresponding condition for the non-error-averse case, obtained from (8), is

\[
\mu_1 \frac{\partial Z_1}{\partial E_1} = \mu_2 \frac{\partial Z_2}{\partial E_2}
\]

Given that \( L'(S) > 0 \) for \( S > 0 \), \( \frac{\partial S}{\partial E_1} > 0 \), and \( \frac{\partial S}{\partial E_2} < 0 \), a positive surprise to \( \phi_1 \) (i.e., \( \phi_1 > \bar{\phi}_1 \)) implies \( \mu_1 \frac{\partial Z_1}{\partial E_1} > \mu_2 \frac{\partial Z_2}{\partial E_2} \) in (11), since the second and third terms on the left-hand side are both
negative. Thus the individual devotes less effort to elaborating commodity 1 – the good that exceeds expectations – and more to elaborating commodity 2 in equilibrium than if he does not experience regret, as evidenced from (12). The finding reflects the compensatory use of elaboration, which competes with its complementary use, and is consistent with the observation above that error aversion engenders a loss in hedonic utility. But, importantly, one also notes that the individual does not employ excess elaboration of commodity 2 to the point that $S = 0$, as that would imply $\mu_1 \frac{\partial Z_1}{\partial E_1} = \mu_2 \frac{\partial Z_2}{\partial E_2}$, a contradiction when elaboration of commodity 2 is set too high relative to the non-error-averse case. Thus, we obtain in equilibrium the additional result that the agent experiences losses due to lingering error regret.

Finally, as Propositions 4 and 5 indicate, a positive surprise to $\phi_1$ in period 1 results in the agent choosing too little of physical good 1 in period 2. This “escalation of commitment” effect reduces utility obtained in the second period relative to what the agent would have obtained were he not affected by error aversion.

PROPOSITION 6: In the event of a surprise about product quality $(\phi_1, \phi_2) \neq (\bar{\phi}_1, \bar{\phi}_2)$, an agent affected by error aversion experiences three sources of welfare loss relative to an agent who is not affected by error aversion:

1. He loses hedonic utility from not elaborating his consumption optimally.
2. He suffers lingering error aversion regret ($S \neq 0$).
3. He loses hedonic utility in future periods from failing to choose an optimal allocation of the physical goods in those periods.
IV. EXTENSION TO STRATEGIC AGENT

TBD

V. CONCLUSION

This paper has provided a new enrichment to the theory of sunk-cost bias and escalation of commitment. For this purpose, it advanced a model of complementary individual action and cognition that, based on the findings of recent research, realistically extends our conception of the consumer decision process. In this context, we showed that whether one escalates commitment to bad choices or instead learns from and moves beyond past errors may be seen as turning on whether one experiences a backward-looking concern for errors made. This is because the way in which people think about what they do differs critically when they experience such regret relative to when they do not. It also has to do with individuals’ reliance on past enjoyment as an indicator of product quality (or, equivalently, of their own preferences).

While ample evidence indicates that how people think about what they do affects their utility from actions, more research needs to be conducted into confirming how individuals perceive their use of elaborative thinking. Are people aware that they do it? Is our assumption of individuals allocating elaborative effort to maximize utility a good approximation of actual behavior? And if individuals do have foresight about the role of elaboration in their utility from consumption, are there specific areas of myopia that affect
their ability to predict its effects? Better information on these issues will allow models with greater predictive power to be developed.

The fact that people engage in elaborative thinking has a range of implications not explored in the paper. We identify two interesting areas where further research could prove useful. The first is the area of conspicuous consumption. It has been argued that social pressures to pursue wealth and consume certain highly-visible goods result in individuals leading lives that, while flush with material things, are devoid of happiness (Scitovsky 1976, Schor 1998, Frank 1999). Incorporating elaboration into consumer behavior modeling offers a way of formally analyzing how “driven” consumption might contribute less utility (i.e., because it is elaborated less), whereby materialistic social drives might result in aggregate welfare loss. The second concerns how people behave when under externally imposed consumption restraints – the closeted gay person, the member of a persecuted religious minority group, the drug user in a rehab program, the teenager prohibited from seeing a significant other, and so forth. The complementarity of elaboration – which is not restricted and potentially unobservable – with restricted physical consumption behaviors may explain dynamic patterns of relapse or “cheating” on constraints in such situations. For example, the act of pining for a fix may make the recovering drug user more likely to relapse; and yet such activities are likely difficult or impossible for rehab centers to monitor and control.

APPENDIX

14 Differences in elaboration potentially related to variations in the extent of conspicuous consumption have been observed anecdotally. Vohs et al. (2013) note that the French heavily ritualize eating (e.g., relative to Americans), an observation that parallels Scitovsky’s observations that Europeans seem to get more pleasure out of their consumption activities than status-driven Americans.
Derivation of Assumption 2.

The expression \( \frac{\partial^2 U}{\partial E_i \partial X_j} \) may be expanded as follows:

\[
\frac{\partial^2 U}{\partial E_i \partial X_j} = \frac{\partial^2 U}{\partial E_i \partial E_i} \frac{\partial E_i}{\partial X_j} + \frac{\partial^2 U}{\partial E_i \partial Z_k} \frac{\partial Z_k}{\partial X_j} + \frac{\partial^2 U}{\partial E_i \partial E_j} X_j + \frac{\partial^2 U}{\partial E_i \partial Z_k} \frac{\partial Z_k}{\partial X_j}
\]

\[
= \mu_i \frac{\partial^2 U}{\partial E_i \partial X_j} + \frac{\partial Z_k}{\partial X_j} \frac{\partial \mu_i}{\partial X_j} \frac{\partial Z_k}{\partial X_j}
\]

\( \frac{\partial^2 U}{\partial E_i \partial X_j} > 0 \) is therefore equivalent to

\[
\mu_i \frac{\partial^2 U}{\partial E_i \partial X_j} > - \frac{\partial Z_k}{\partial X_j} \frac{\partial \mu_i}{\partial X_j} \frac{\partial Z_k}{\partial X_j} \iff \frac{\partial^2 U}{\partial E_i \partial X_j} > \frac{\partial Z_k}{\partial X_j} \frac{\partial \mu_i}{\partial X_j} \frac{\partial Z_k}{\partial X_j}
\]

\[
\iff \eta_{E_i} > \left| \eta_{Z_k} \right| \iff \eta_{E_i} > \left| \eta_{Z_k} \right|
\]

Proof of Proposition 1.

The second-order conditions may be expressed as the determinant of the bordered Hessian. To wit,

\[
|H| = \begin{vmatrix}
\mu_i \frac{\partial^2 Z_k}{\partial E_i \partial E_i} + \frac{\partial \mu_i}{\partial E_i} \left( \frac{\partial Z_k}{\partial E_i} \right)^2 & \frac{\partial \mu_i}{\partial Z_k} \frac{\partial Z_k}{\partial E_i} & 1 \\
\frac{\partial \mu_i}{\partial Z_k} \frac{\partial Z_k}{\partial E_i} & \mu_i \frac{\partial^2 Z_k}{\partial E_i \partial E_i} + \frac{\partial \mu_i}{\partial E_i} \left( \frac{\partial Z_k}{\partial E_i} \right)^2 & 1 \\
1 & 1 & 0
\end{vmatrix}
\]

\[
= \frac{\partial \mu_i}{\partial E_i} \left( \frac{\partial Z_k}{\partial E_i} \right)^2 - \mu_i \frac{\partial^2 Z_k}{\partial E_i \partial E_i} \left( \frac{\partial Z_k}{\partial E_i} \right)^2 - \mu_i \frac{\partial^2 Z_k}{\partial E_i \partial E_i} \left( \frac{\partial Z_k}{\partial E_i} \right)^2 + \frac{\partial \mu_i}{\partial E_i} \left( \frac{\partial Z_k}{\partial E_i} \right)^2 + \frac{\partial \mu_i}{\partial Z_k} \frac{\partial Z_k}{\partial E_i} - \frac{\partial \mu_i}{\partial Z_k} \frac{\partial Z_k}{\partial E_i} - \frac{\partial \mu_i}{\partial Z_k} \frac{\partial Z_k}{\partial E_i} - \mu_i \frac{\partial^2 Z_k}{\partial E_i \partial E_i} - \mu_i \frac{\partial^2 Z_k}{\partial E_i \partial E_i}
\]

The key to identifying the solution to the first-order conditions as a unique global maximum is to sign this expression as globally positive. Since the last two terms are positive by our assumptions about the function \( Z_i \), it will be sufficient to sign the expressions in brackets as nonnegative. Each of these represents the manner in which the marginal utility of effort with respect to a commodity changes when effort is allocated at the margin away from elaboration of that commodity to elaboration of the other commodity. The expressions are
clearly positive for complements and independent goods. For substitutes, the terms on both sides of the minus sign are negative, but the first term is never larger than the second; in the extreme case of perfect substitutes, the terms are equal. Thus the determinant of the bordered Hessian is globally positive. The first part of the Proposition follows.

Now, using Cramer’s rule,

\[
\frac{\partial E_1}{\partial \phi_1} = \frac{1}{|H|} \begin{vmatrix}
-\frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_1}{\partial E_1} & \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_2}{\partial E_1} \\
0 & \mu_2 \frac{\partial^2 z_2}{\partial E_1^2} + \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial z_2}{\partial E_1} \right)^2 \\
0 & 1
\end{vmatrix} = \frac{1}{|H|} \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_1}{\partial E_1} > 0
\]

which yields the second part of the Proposition.

**Proof of Proposition 2.**

The second-order conditions may be expressed

\[
|H| = \begin{vmatrix}
\mu_2 \frac{\partial^2 z_2}{\partial E_1^2} + \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial z_2}{\partial E_1} \right)^2 & \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_1}{\partial E_1} \\
-\mu_2 (S) \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial S}{\partial z_2} \right) & -\omega L''(S) \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial S}{\partial z_2} \right) - \omega L'(S) \frac{\partial^2 S}{\partial E_1 \partial E_2} \\
\frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_1}{\partial E_1} & \mu_2 \frac{\partial^2 z_2}{\partial E_2^2} + \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial z_2}{\partial E_2} \right)^2 \\
-\omega L''(S) \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial S}{\partial z_1} \right) & -\omega L''(S) \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial S}{\partial z_1} \right) - \omega L'(S) \frac{\partial^2 S}{\partial E_2 \partial E_2} \\
1 & 1
\end{vmatrix}
\]

which simplifies to

\[
|H| = \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_1}{\partial E_1} \left[ \mu_2 \frac{\partial^2 z_2}{\partial E_1^2} + \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial z_2}{\partial E_1} \right)^2 \right] + \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_2}{\partial E_1} \left[ \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_1}{\partial E_1} \right] + \left[ \mu_2 \frac{\partial^2 z_2}{\partial E_2^2} + \frac{\partial \mu_1}{\partial \phi_1} \left( \frac{\partial z_2}{\partial E_2} \right)^2 \right]
\]

\[
+\omega L''(S) \left( \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial S}{\partial E_1} - \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial S}{\partial z_1} \right) + \omega L'(S) \left( \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial S}{\partial E_2} + \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_2}{\partial E_2} \right)
\]

Given this, the necessary and sufficient condition for a maximum may be written

(A1) \[\omega L'(S) \left( \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial S}{\partial E_2} + \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_2}{\partial E_2} \right) + \omega L''(S) \left( \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial S}{\partial E_2} - \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial z_2}{\partial E_2} \right) > -\left| H \right| - \omega \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial S}{\partial \phi_0}\]
where

$$\begin{align*}
|H| &= -\omega \frac{\partial^2}{\partial \omega} = \frac{\partial Z_1}{\partial E_1} \frac{\partial Z_1}{\partial E_2} + \left[ \mu_1 \frac{\partial Z_1}{\partial E_1} \mu_2 \left( \frac{\partial Z_2}{\partial E_2} \right)^2 \right] \\
(A2) &+ \frac{\partial Z_2}{\partial E_1} \frac{\partial Z_2}{\partial E_2} - \left[ \mu_1 \frac{\partial Z_2}{\partial E_1} \left( \frac{\partial Z_2}{\partial E_2} \right)^2 \right] > 0
\end{align*}$$

is the Hessian from the non-error-averse case.

The relevant derivatives of $S$ are

$$
\begin{align*}
\frac{\partial S}{\partial \phi_1} &= \frac{\partial \mu_1}{\partial \phi_1} \frac{\partial Z_1}{\partial \phi_1} \frac{\partial Z_1}{\partial X_1} > 0 ; \quad \frac{\partial S}{\partial \phi_2} = -\frac{\partial \mu_2}{\partial \phi_2} \frac{\partial Z_2}{\partial \phi_2} \frac{\partial Z_2}{\partial X_2} < 0 \\
\frac{\partial S}{\partial E_1} &= \frac{\partial \mu_1}{\partial E_1} \frac{\partial Z_1}{\partial E_1} \frac{\partial Z_1}{\partial E_2} + \mu_1 \frac{\partial Z_1}{\partial E_2} \frac{\partial Z_1}{\partial X_1} \frac{\partial Z_1}{\partial X_2} \\
\frac{\partial S}{\partial E_2} &= \frac{\partial \mu_1}{\partial E_2} \frac{\partial Z_2}{\partial E_1} \frac{\partial Z_2}{\partial E_2} + \mu_2 \frac{\partial Z_2}{\partial E_2} \frac{\partial Z_2}{\partial X_1} \frac{\partial Z_2}{\partial X_2}
\end{align*}
$$

REMARK 2. $\frac{\partial S}{\partial \phi_1}$ is continuous in $\phi_1$ on $[0,1]$. $\frac{\partial S}{\partial \phi_2}$ is continuous in $\phi_2$ on $[0,1]$.

It can be seen from (A1) and (A2) that the necessary and sufficient conditions for a maximum are met for $S$ sufficiently close to zero. Moreover, Remarks 1 and 2 imply that a neighborhood of $(\bar{\phi}_1, \bar{\phi}_2)$ exists such that $S$ on that neighborhood is sufficiently close to zero to meet the conditions for a maximum. The first part of the Proposition follows.

We now turn to results needed for the second part of the Proposition.

LEMMA 1. $\frac{\partial S}{\partial E_1} > 0$ and $\frac{\partial S}{\partial E_2} < 0$.

The lemma follows directly from Assumptions 1 and 2.

Using Cramer’s rule,
\[
\frac{\partial E_i}{\partial \phi_i} = \frac{1}{|H|} \begin{bmatrix}
\omega L''(S) \frac{\partial^2 S_i}{\partial \phi_i^2} + \omega L''(S) \frac{\partial S_i}{\partial \phi_i} \frac{\partial S_i}{\partial \phi_i} - \omega L''(S) \frac{\partial S_i}{\partial \phi_i} \frac{\partial S_i}{\partial \phi_i} - \omega L'(S) \frac{\partial^2 S_i}{\partial \phi_i^2} & 1 \\
\omega L''(S) \frac{\partial S_i}{\partial \phi_i} \frac{\partial S_i}{\partial \phi_i} & 0 \\
0 & 1 \\
\end{bmatrix} \begin{bmatrix}
\mu_2 \frac{\partial^2 S_i}{\partial \phi_i^2} + \frac{\partial \mu_2}{\partial \phi_i} \left( \frac{\partial S_i}{\partial \phi_i} \right)^2 \\
\omega L''(S) \left( \frac{\partial S_i}{\partial \phi_i} \right)^2 - \omega L'(S) \frac{\partial^2 S_i}{\partial \phi_i^2} \\
0 & 0 \\
\end{bmatrix}
\]

Differentiation with respect to \( \omega \) yields

\[
\frac{\partial^2 E_i}{\partial \phi_i \partial \omega} = \frac{1}{|H|^2} \left[ |H| \left[ L''(S) \frac{\partial S_i}{\partial \phi_i} \frac{\partial S_i}{\partial \phi_i} - L'(S) \frac{\partial^2 S_i}{\partial \phi_i^2} \right] \frac{\partial |H|}{\partial \omega} - \omega \frac{\partial |H|}{\partial \omega} \left[ L''(S) \frac{\partial S_i}{\partial \phi_i} \frac{\partial S_i}{\partial \phi_i} - L'(S) \frac{\partial^2 S_i}{\partial \phi_i^2} \right] \right]
\]

Given Lemma 1, \( S=0 \) implies \( \frac{\partial^2 E_i}{\partial \phi_i \partial \omega} < 0 \). Thus there exists a neighborhood of \( S=0 \) in \( \mathbb{R} \) in which \( \frac{\partial^2 E_i}{\partial \phi_i \partial \omega} < 0 \) holds. By Remarks 1 and 2, then, it follows that there also exists a neighborhood of \((\tilde{\phi}_1, \tilde{\phi}_2)\) such that \( \frac{\partial^2 E_i}{\partial \phi_i \partial \omega} < 0 \) holds.

\[\text{Proof of Propositions 3 and 4.}\]
As noted previously, if $\phi_1, \phi_2$ were known, physical goods quantities would be chosen such that $S$ would equal 0. Because $\phi_1, \phi_2$ are not known, as assumed above the individual conjectures $(\phi_1, \phi_2) = (\bar{\phi}_1, \bar{\phi}_2)$, which implies

REMARK 1. $S = 0$ for $(\phi_1, \phi_2) = (\bar{\phi}_1, \bar{\phi}_2)$.

Let us initially suppose that $(\phi_1, \phi_2) = (\bar{\phi}_1, \bar{\phi}_2)$. It follows from Remark 1 that $S = 0$, whence utility maximization using (4) and (6) achieve equivalent results. This entails, inter alia, equivalent choices of $E^*_i$, so conjectures about $\phi_i$ remain correct.

LEMMA 2. $\forall \omega, (\hat{\phi}_1, \hat{\phi}_2) = (\bar{\phi}_1, \bar{\phi}_2)$ when $(\phi_1, \phi_2) = (\bar{\phi}_1, \bar{\phi}_2)$.

The next step is to examine the effect of $\omega > 0$ on conjectures about $\phi_i$ more generally. When $\omega = 0$, as Proposition 3 indicates, conjectures vary with the actual $\phi_i$ “one-for-one,” that is, $\frac{\partial \hat{\phi}_i}{\partial \phi_i} = 1$. Proposition 2 indicates that $\frac{\partial^2 E_i}{\partial \phi_i \partial \omega} < 0$; however the individual in this case continues to presume variation in the $E_i$ consistent with $\omega = 0$ and “reverse-engineers” $\phi_i$ accordingly. Thus natural biases are introduced into the individual’s conjectures; these biases grow with $\omega$, to wit,

LEMMA 3. $\frac{\partial^2 E_i}{\partial \omega \partial \phi_i} < 0$ implies $\frac{\partial^2 \hat{\phi}_i}{\partial \omega \partial \phi_i} < 0$.
The proposition follows from Lemmas 2 and 3 and Proposition 3. For the corollary, we not that by assumption, $\frac{d^2 \pi}{d \alpha \partial \phi} > 0$. This means biased conjectures $\hat{\phi}_i$ bias the marginal utility of the commodity in the same direction. It follows that corresponding choices $X_i$ are similarly, and to a commensurate extent, biased.

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