Advertising and Self-Persuasion

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Abstract

In a world in which consumers can improve their attitudes towards prospective choices, I examine the market price implications of advertising that facilitates self-persuasion efforts. Consistent with existing advertising theories, advertising raises prices, all else equal, to the extent its facilitation effects fall selectively on infra-marginal consumers and lowers them to the extent they fall on marginal consumers. But advertising also brings pre-existing patterns of consumer self-persuasion to fuller expression, manifesting the price effects those patterns represent. My model delivers improved predictions of advertising's effects compared to approaches that flatly conceive of advertising as increasing the consumer's willingness-to-pay.

Keywords Persuasive advertising, prices, targeting, motivated preferences

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"It's good to want things."

- Winona Ryder (as Dinky Bossetti) in *Welcome Home, Roxy Carmichael.*

1 Introduction

A substantial amount of money is spent by firms every year on advertising. In 2021, global advertising expenditures were approximately $723 billion (Houston 2022). Clearly not all of this advertising provides hard information. One would be hard pressed to argue, for example, that most television commercials for beer are focused on providing pertinent facts about the product's tangible characteristics. To be sure, even advertising messages that appear non-substantive (e.g., image-oriented) convey the information that the product is advertised and therefore must be of sufficient quality to have elicited a costly advertising expenditure (Nelson 1974, Kihlstrom and Riordan 1984). Yet this alone cannot explain costly efforts devoted to crafting messages and images in ads otherwise devoid of informational content. If the purpose of an ad is just to show that money is being spent and the ad is not intended in some measure to be persuasive, why should such details matter? And if signaling expenditures is the main purpose of such advertising, then why do we not observe companies advertising how much they spend on advertising (Becker and Murphy 1993)?

On the other hand, no theory has offered an explanation as to why, and by what process, purely persuasive messages elicit a response from a consumer - except in cases in which the consumer erroneously processes such messages as information (see Section 2 below). It is typical of existing theories to describe non-informative advertising as increasing the targeted consumer's willingness to pay (WTP) for the product advertised without seeking to specify the process internal to the consumer that is responsible for that increase. But can a theory based on this spare presumption, and agnostic to its underpinnings, accurately predict the price effects of persuasive advertising?

This article, which investigates the market price implications of advertising, advances an explicit proposition as to how persuasive advertising works: it facilitates consumers' efforts to persuade themselves. Following Nagler (2021, 2022), I posit a consumer who fabricates cognitions that complement the consumption of products that he has chosen or contemplates choosing. For example, a consumer thinking of purchasing a certain brand of beer dials up ideas, images, and arguments (e.g., this beer has a casual image, my friends will respect me more if I drink this brand of beer) that support the choice. These fabrications, which focus on select characteristics of the product, increase the utility the consumer will obtain from consumption. Such manipulations of utility are made possible by a form of bounded rationality in which consumption utility depends on what is salient to the consumer, an approach consistent with a focus-weighted model of utility (Kőszegi and Szeidl 2013) and underpinned by psychological evidence that top-of-mind phenomena exert an outsized influence on attitudes (Taylor and Fiske 1978, Tesser 1978, Taylor et al. 1979, Borgida and Howard-Pitney 1983). Broadly speaking, consumers are able to reorganize their ideas about objects in order to make certain attributes more influential on their perceptions and experiences. And it is rational for a consumer to engage in such reorganization because, as the quote at the start of the paper puts it, it is good to want things.

Coming up with effective complementary cognitions, however, is difficult. Persuasive advertising in my framework aids the consumer by suggesting ideas, images and arguments that reduce the "cost" of "having" the cognitions. Note that, in my framework, the individual is conceived not as consuming the advertising per se, but the cognitions, which the advertising makes easier. Thus the self-persuasive cognitions – and not the advertising – are the focal element of the approach, and the costs of these cognitions to consumers are its essential primitive.

Psychological evidence points to a self-persuasion mechanism in choice. Independent of external stimuli, individuals have been found to increase the subjective ratings they assign to objects after they have chosen them (Lieberman et al. 2001, Kitayama et al. 2004, Sharot et al. 2010, Wakslak 2012). The adoption of choices has been found moreover to be accompanied by neural markers for preference change (Sharot et al. 2009, Van Veen et al. 2009, Izuma et al. 2010, Jarcho et al. 2011, Qin et al. 2011, Kitayama et al. 2013, Izuma and Adolphs 2013). Notably, recent evidence confirms that choice-induced preference change is integral to the decision process rather than being purely an adjunct of post-choice dissonance reduction (Voigt et al. 2019, Lee and Daunizeau 2020). Individuals think episodically about the future use of chosen objects, searching for positive distinctive features in these objects that might assure a positive experience and seal the choice (Kitayama and Tompson 2015).

Meanwhile, evidence on consumers' response to advertising coheres with framing persuasive advertising conceptually not as an intervention that shifts the preferences of passive consumers, but rather as a resource utilized through active consumer efforts to shift preferences in the face of commitment to choices (Ehrlich et al. 1957, Mills 1965, Tuchman et al. 2015). In this process, message content appears to be recruited selectively by individuals to support and reinforce existing preference positions (Keller and Block 1999, Jain and Maheswaran 2000, Baysan 2022).

Main results

My model incorporates self-persuasion into a Hotelling (1929) spatial competition framework through a marginal cost to the consumer of "relocating" nearer the product. This cost varies across consumers. Advertising reduces marginal self-persuasion costs for the consumers it targets, such that its effect is to improve the technology of self-persuasion heterogeneously across consumers.

I find that the effect of advertising on prices depends in part on how the advertising is targeted, that is, whether it primarily improves the self-persuasion technology of marginal or inframarginal consumers – what I call a "shaping effect." When advertising targets marginal consumers, prices are lower, all else equal, whereas they tend to be higher when advertising targets inframarginal consumers. This outcome is consistent with the predictions of prior work that posits persuasive advertising simply as increasing the consumer's WTP. But advertising's price effect under my model also depends on how the pre-existing technology of self-persuasion is distributed across consumers – specifically whether, independent of the advertising, marginal consumers find it relatively more or less costly to self-persuade with respect to their chosen product than inframarginal consumers. Functionally, this follows because a general effect of advertising that facilitates self-persuasion is to further manifest the existing pattern of relative proclivity for self-persuasion. In so doing, advertising causes prices to rise or fall predictably based on that pattern, in potential conflict with the shaping effect associated with targeting of the advertising. If one ignores the manifestation component, one risks making erroneous predictions of the price effects of particular advertising strategies. Conversely, one may incorrectly deduce that a certain observed price effect was generated by a particular advertising strategy as opposed to a different strategy.

An analogy for the problem of predicting the price effects of advertising in this context is that of predicting the impact of a public policy program. Consider a program that introduces tax-deferred college savings plans. Suppose these plans are targeted at low-income families but also made available to other families. Suppose further that higher-income families have a greater pre-existing marginal propensity to take advantage of college savings opportunities. The net result of the program may be that college saving becomes more income-stratified rather than less after the savings plans are introduced. Purely on the basis of targeting, one would predict that the program would reduce the stratification of college saving. But such a prediction ignores how, as one of its effects, the program more fully manifests existing proclivities for saving. To get the desired profile of benefit, policy makers must recognize the existing proclivities of different groups and adjust their strategies to take these into account. Along similar lines, in predicting the price effects of advertising that aids self-persuasion, it is necessary to account for the pre-existing self-persuasion capabilities of different groups of consumers.

Article structure

Section 2 reviews the related literature. Section 3 lays out the self-persuasion model. Section 4 derives the equilibrium and the results of the model as relates to prices. Section 5 explores the implications of my framework for the empirical approach that models advertising simply as increasing the WTP of targeted consumers. Section 6 concludes. The printed appendix contains proofs and derivations of all lemmas and propositions. An online appendix extends the model's results to a generalized distribution of consumers, and it generalizes the equilibrium results to all continuous convex specifications of self-persuasion.

2 Related literature

The traditional school of thought on persuasive advertising has presumed that it alters consumers' tastes, such that advertising can be modeled as shifting demand for the advertised product (e.g., Dixit and Norman 1978). Core propositions include that advertising creates spurious product differentiation and brand loyalty and so decreases the elasticity of demand for the product, resulting in increased prices and profits (Bain 1956, Comanor and Wilson 1974, Bagwell 2007). The theory is largely silent on the primitive basis for its assumptions about effects on product perceptions and brand preference.

In a significant conceptual advance, Becker and Murphy (1993) propose that advertising should be modeled as a complement to the product advertised. Advertising proffers images and persuasive messages that the individual in effect consumes along with the product. This act of consuming increases the marginal utility of the product for those consumers for whom the images and messages are influential. While this process naturally raises the WTP of those consumers, Becker and Murphy point out that it does not necessarily increase the market equilibrium price: the effect on price depends on which consumers' WTP are affected and the consequent implications for the elasticity of market demand. Becker and Murphy's presumption that the complementarity of advertising and product leads to increases in consumer WTP has been cited as motivation for modeling advertising empirically as increasing the WTP of targeted consumers (Ackerberg 2001, Erdem et al. 2008). More broadly, numerous papers employ empirical models in which advertising is posited to increase consumers' WTP (e.g., Snyder and DeBono 1985, Hampel et al. 2012, Wang et al. 2018).

A missing element in Becker and Murphy's theory is that it proposes no basis for the expectation that persuasive elements in advertisements should influence the WTP of a rational consumer. While their complementary concept has an intuitive appeal, their proposition is based on an under-specified primitive, in that they do not model, or even identify, the mechanism inside the consumer's mind that catalyzes the advertising content's complementarity with consumption of the advertised product.[1] This turns out to be problematic for making accurate predictions about advertising's effects on prices, as I detail in Section 5 with reference to the approach presented in this article.

Several other articles relate closely to the present work. Mullainathan et al. (2008) describe a process – "coarse thinking" – according to which persuasive content is processed erroneously as information, rendering it effective at influencing a boundedly rational consumer's decision. Along roughly similar lines, Salant and Siegel (2018) propose that consumers are susceptible to persuasive "framing." My paper sees the consumer's responsiveness to persuasion, in contrast, as arising out a rational motivation to achieve the desired goal of optimal preference adjustment. Such consumer use of persuasive content is consistent with the observed behavior of the experimental participants in Ehrlich et al. (1957) and Mills (1965), as well as the empirical findings of Tuchman et al. (2015).

Like the present article, Bloch and Manceau (1999) use a Hotelling model to analyze the effect of persuasive advertising. Advertising in their model shifts the preference

[1]Why, for example, should "associations between sexual allure and the products advertised" (Becker and Murphy 1993, p943) increase the expected utility of product consumption for a consumer who surely understands, on a purely factual basis, that the beer for sale is not bundled with a promise of sexual favors? To be sure, one may enjoy the sexual content of the ad so designed, by why would one enjoy the advertised product any more for having watched it, as opposed to whatever product (e.g., competing beer) one happens to be consuming while watching the ad?

distribution of consumers closer to the advertiser's product, based mathematically on the principle of stochastic dominance. Thus, consistent with much of the literature on persuasive advertising, they show advertising to be as a zero-sum game of demand shifting between competitors. In contrast with this, advertising by each firm in my model unambiguously creates surplus in the form of reduced self-persuasion costs for consumers, whence advertising competition results in additive benefits rather than a wasteful tug-of-war. (Bloch and Manceau do not explain on what basis advertising in their model is predicted to shift consumers' preferences.)

Haan and Moraga-González (2011) and Astorne-Figari et al. (2019) conceive of consumers as boundedly rational, whereby firms use advertising to vie for the consumer's limited attention and powers of recall. Unlike the present article, they do not conceive of advertising as persuasive, but effectively informative: advertising reminds a consumer who is forgetful - in the sense of not having essential information accessible at top of mind - of the availability of certain products. Eliaz and Spiegler (2011) also propose that consumers are boundedly rational in the traditional sense, but conceive of advertising as persuading the consumer to include the advertised product in his limited consideration set. Thus their theory of "advertising as persuasion" is novel in that it maintains that preferences are stable. But while the goal of the advertiser in Eliaz and Spiegler's theory may be ostensibly to persuade, what advertising boils down to is information to be used to make a decision as to whether the product should be considered.

The idea of advertising as a tool utilized by the consumer has several precedents in the literature. It is consistent with the Elaboration Likelihood Model, which considers the conditions under which consumers approach advertising with active thinking as a part of their decision-making process (Petty and Cacioppo 1986); and it is a feature of the uses and gratifications literature (O'Donohoe 1994, Ko et al. 2005, Aitken et al. 2008, Phillips and McQuarrie 2010). Advertising may be viewed as a stand-in for other forms of costly marketing communication, particularly salesperson communications to customers, which have been discussed as facilitating consumers' efforts at post-purchase dissonance reduction (Hunt 1970, Milliman and Decker 1990, Grewal and Sharma 1991).

3 A model of self-persuasion and facilitative advertising

Consumers, preferences, and self-persuasion

Consider two differentiated products, indexed by $j \in \{0,1\}$, each produced by an independent firm correspondingly named. Following Nagler (2021), let us suppose that consumers' preferences over these products have two components. Each consumer is endowed with an *initial* preference, also known as his *non-motivated* component, which may be viewed as an exogenous characteristic of the individual. Each consumer's preference also comprises a *motivated* component developed through a process of *self-persuasion*. This process entails costly effort by the consumer to increase the satisfaction he expects to receive from consuming the object he has chosen, where the costs - like the consumer's initial preference - are an exogenous characteristic of the individual. Consistent with a focus-weighted conception of utility (Kőszegi and Szeidl 2013), self-persuasion may be thought of as involving efforts to maintain as salient in the mind a characterization of the product that allows the consumer to be more satisfied with his presumed choice, based on reorganizing information already in memory and combining this information with instrumentally-sought external cues. The consumer knows his costs of self-persuasion and fully anticipates the benefits to be gained from his efforts.

The assumptions are formalized using a Hotelling spatial model. The two firms are located at opposite ends of a segment of length 1 representing the product space. A consumer base with a measure of one is distributed uniformly on a subsegment $[\nu, 1-\nu]$ of the product space for $\nu \in \left[0, \frac{1}{2}\right]$; here, $\nu = 0$ generates as a special case the typical full-unit-segment uniform distribution of consumers, while $\nu = \frac{1}{2}$ generates a polar case characterized by maximum concentration at the center. Intermediate levels of ν allow for variations in the central tendency of the consumer distribution.[2] The consumer's location $x \in [\nu, 1-\nu]$ identifies his initial relative preference over the two products. Consumers buy at most one unit of a single product, so based on the non-motivated component alone the utility of a consumer at x buying product j is given by

$$U_x = V - p_j - t\,|x - j| \tag{1}$$

[2]In the online appendix, I show that, subject to some minor restrictions, the main results of the model still hold for a fully general consumer density f on $[0,1]$.

where V is the common reservation price for the product, p_j is the price of product j, and t parameterizes the utility loss due to j's not being the consumer's ideal choice – the standard "transportation" cost, linear in the consumer's distance from j. All consumers have an outside option of utility zero.

Self-persuasion in the spatial context consists in relocating on the segment to be closer to the product's location, thereby paying less transportation cost. A marginal self-persuasion cost (MSPC) function represents the cost of the effort required for self-persuasion associated with product j by the consumer at x: $g^j(i,x) > 0$, continuous in x and i, and monotone increasing and convex in i with $\lim_{i\to x} g^j(i,x) = \infty$, where i is the consumer's attitude improvement, or distance traveled from x toward j's position.[3] Continuity implies that self-persuasion effort is smooth for a given consumer and its variation from one consumer to the next along the continuum of consumers is smooth. Monotonicity and convexity reflect incremental self-persuasion becoming progressively more difficult as the best opportunities for taking a more positive perspective on the product are used up. Growth without limit implies that no matter how positive one's attitude toward a product, it is possible to further improve that attitude, such that a consumer never fully converges to the product's location even as he continues to work at self-persuasion.

One may represent the MSPC function by a set of self-persuasion *curves* $\mathscr{G}^j := \{g^j(i) = g^j(i,x) : x \in [\nu, 1-\nu]\}$ characterized by differing values of x, whereby each curve represents the cost, at each state of i, of incremental "movement toward" j for the consumer located initially at x. I shall refer to $\mathscr{G}^j$ as a self-persuasion *map* for product j. Figure 1 illustrates a possible self-persuasion map for product 0.

Advertising

Advertising for a given product is conceived as making self-persuasion easier with respect to that product, that is, improving the consumer's product-specific technology of self-persuasion so that the same amount of attitude improvement toward the product may be achieved at lower cost. I represent this by specifying an advertising-composite MSPC

[3] This representation invites the interpretation of a consumer producing a complement to the product that resides internally: the MAC is the marginal cost to the consumer of producing the complement, while the marginal utility of the complement is -1 times the transportation cost t - that is, the complement's opportunity cost.

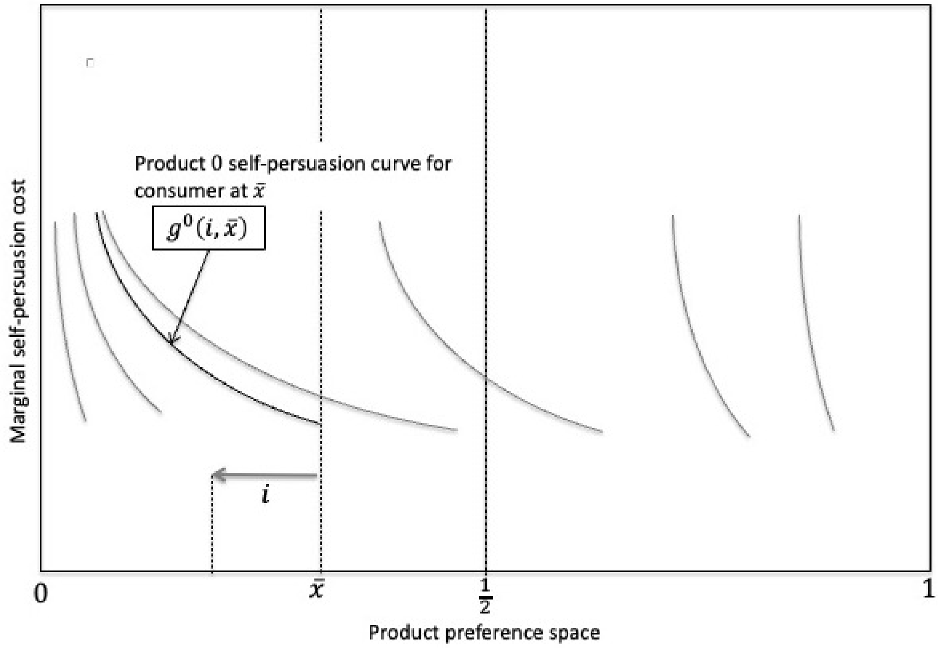

Figure 1: A Self-Persuasion Map (example for $\nu = 0$)

function that weights pre- and post-advertising MSPC component functions,

$$g^j(i, x, A_j) = \phi(A_j) g^{j,1}(i, x) + [1 - \phi(A_j)] g^{j,0}(i, x) \tag{2}$$

where A_j represents firm j's advertising expenditure; the $g^{j,k}(i,x)$ for $k = 1, 2$ observe the characteristics described for the $g^j(i,x)$ above; and $g^{j,0}(i,x) > g^{j,1}(i,x) > 0$ $\forall x, i$. The advertising impact function $\phi(.)$ is continuous, increasing, and strictly concave on its support, with $\phi(0) = 0$ and $\lim_{A_j \to \infty} \phi(A_j) = 1$. Advertising under this general specification causes product-specific MSPCs to converge asymptotically from a zero-advertising level given by $g^{j,0}(i,x)$ toward $g^{j,1}(i,x)$ as illustrated (for $j = 0$) in Figure 2. Note also that the specification allows for advertising's effects to vary in a fully general way across individual consumers.

Parameterization

For the purposes of the remaining analysis,[4] I propose the following parameterization for the MSPC components in (2):

$$\begin{aligned} g^{0,0}(i,x) &= \frac{b}{1-\frac{i}{x}} - (1-2x)\,\sigma_0 \text{ for } x \in [0,1],\ i \in [0,x)\ ; \\ g^{1,0}(i,x) &= \frac{b}{1-\frac{i}{1-x}} - (2x-1)\,\sigma_0 \text{ for } x \in [0,1],\ i \in [0,1-x)\ ; \\ g^{0,1}(i,x) &= \frac{b}{1-\frac{i}{x}} - (1-2x)\,\sigma_1 - \theta \text{ for } x \in [0,1],\ i \in [0,x)\ ; \\ g^{1,1}(i,x) &= \frac{b}{1-\frac{i}{1-x}} - (2x-1)\,\sigma_1 - \theta \text{ for } x \in [0,1],\ i \in [0,1-x)\ . \end{aligned} \tag{3}$$

such that

$$\begin{aligned} g^{0}(i,x,A_0) &= \frac{b}{1-\frac{i}{x}} - (1-2x)\,\sigma_0 - \phi(A_0)\left[(1-2x)(\sigma_1-\sigma_0)+\theta\right] \\ g^{1}(i,x,A_1) &= \frac{b}{1-\frac{i}{1-x}} - (2x-1)\,\sigma_0 - \phi(A_1)\left[(2x-1)(\sigma_1-\sigma_0)+\theta\right] \end{aligned} \tag{4}$$

for $\sigma_0, \sigma_1, \theta \geq 0$ and $b > 0$.

The parameters have intuitive interpretations as regards the effects of self-persuasion-facilitating advertising. θ parameterizes the relative size of advertising's mass-market effect - that is, its relative role as a *general* facilitator of self-persuasion. The difference $\sigma_1 - \sigma_0$ measures the targeting of the advertising: $\sigma_1 - \sigma_0 > 0$ indicates advertising that singles out inframarginal (i.e., loyal or strong-preferenced) consumers, $\sigma_1 - \sigma_0 < 0$ advertising focused on marginal (i.e., uncommitted or weak-preferenced) consumers. The relative magnitudes of θ and $\sigma_1 - \sigma_0$ pick up the relative size of advertising's mass-market effect and targeted effects, respectively. The incorporation of these effects as exogenous parameters is consistent with the advertising literature. Much of the literature treats the decision of whether to target marginal consumers or inframarginal consumers as determined by characteristics of the product or brand, such as whether the brand is an established brand (e.g., Graham and Kennedy 2022), or whether the product can be effectively differentiated based on horizontal attributes (e.g., Erdem et al. 2008).

The model parameters also create a structure for describing how self-persuasion ca-

[4] In the online appendix, I show that the main results of the model follow when a nonparametric approach is pursued using a general g.

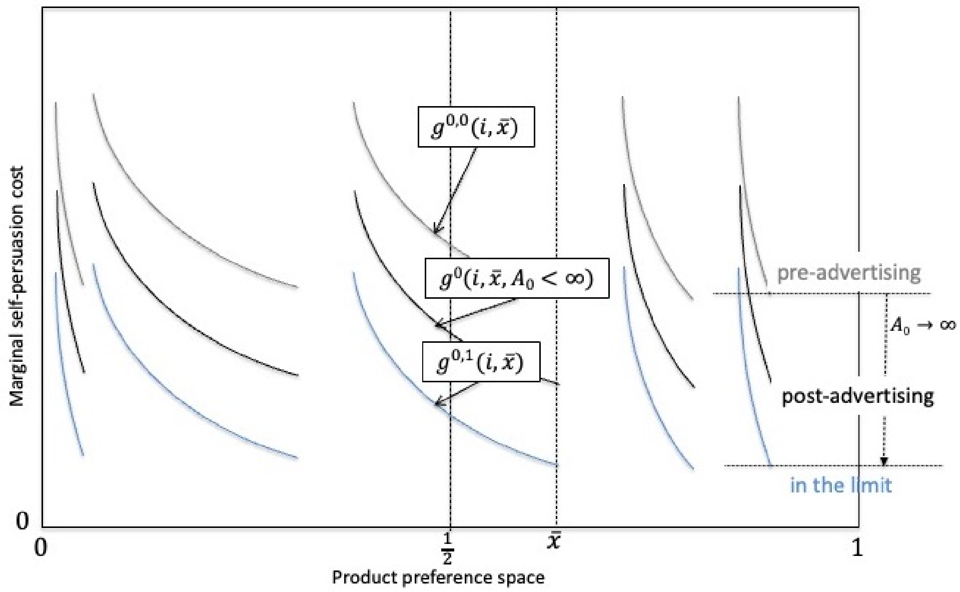

Figure 2: Pre- and Post-Advertising Self-Persuasion Maps

pabilities vary systematically across consumers, pre- and post-advertising. The structure is defined in terms of the *mean-regressivity* of self-persuasion: the degree to which consumers whose non-motivated preference for a product is weaker have a relative advantage at self-persuasion, whereby they see their preferences converge toward those consumers with stronger preferences once the motivated component of preference is added in. σ_0 parameterizes the *pre-advertising level* of self-persuasion mean-regressivity and σ_1 the *post-advertising limiting level* (i.e., for $\phi(A_j) = 1$). The baseline pattern - i.e., with $\sigma_0 = \sigma_1 = 0$ - is maximally mean-regressive: it involves an initial (i.e., at $i = 0$) MSPC for all consumers equal to b pre-advertising and $b - \theta$ post-advertising. Higher values of σ_0 and σ_1 correspond to a reduced mean-regressivity of self-persuasion pre- and post-advertising, respectively: specifically, they imply an initial MSPC that declines linearly with the proximity of the consumer's initial position x to the product - and so imply diminishing self-persuasion advantages for consumers with weaker initial preferences relative to $\sigma_k = 0$, $k = 0, 1$. Figure 3 illustrates both the baseline $\sigma_k = 0$ case and the case for $\sigma_k > 0$. Lower relative levels of mean-regressivity indicate initial impressions play an important role in final preferences, such that relative preferences across consumers are more strongly path dependent rather than being subject to convergence through effort. Advertising that targets inframarginal consumers makes self-persuasion less mean-regressive, while advertising that targets marginal consumers makes it more

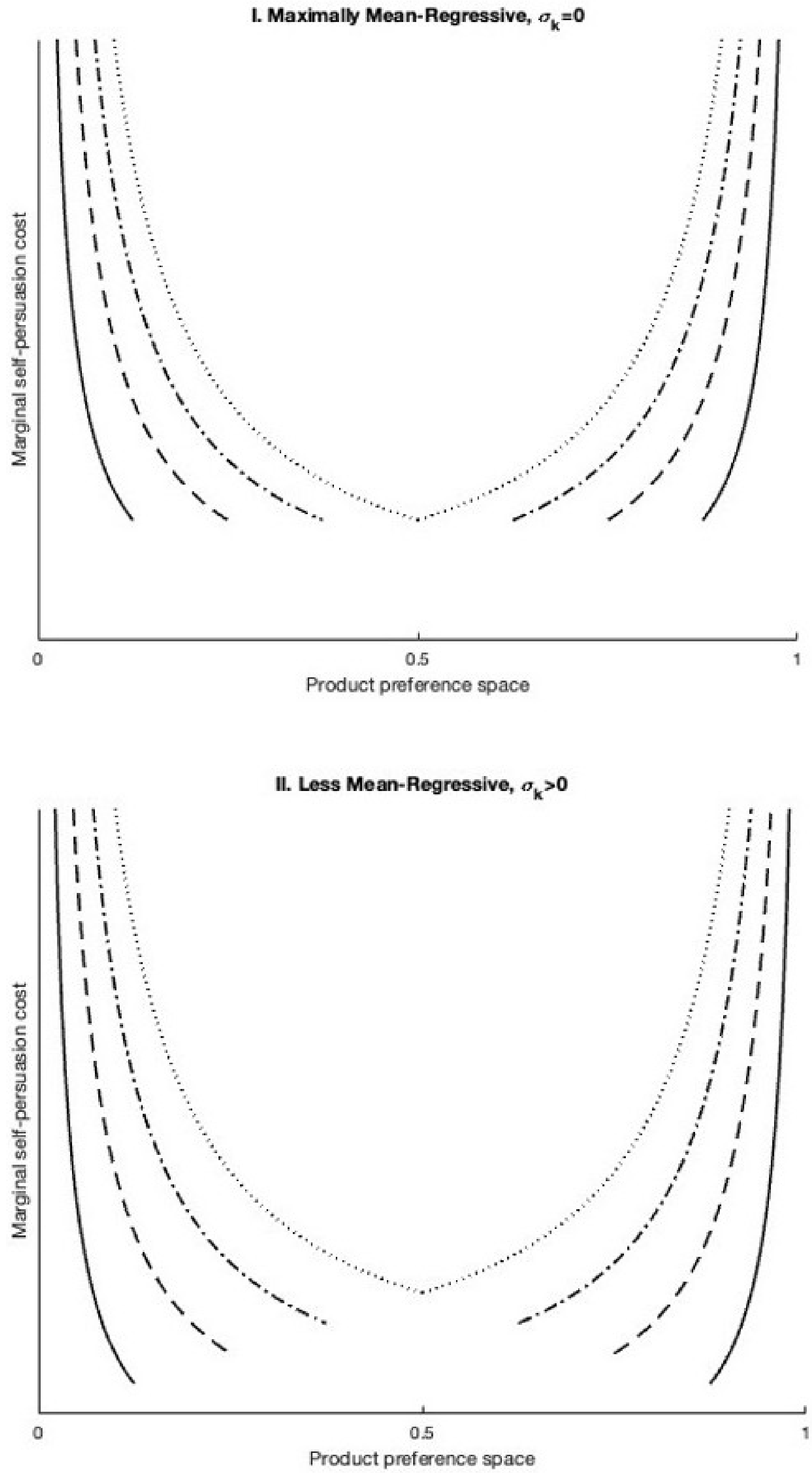

Figure 3: Self-persuasion maps for $\sigma_k = 0$ vs. $\sigma_k > 0$

mean-regressive.

The specification in (4) imposes two main restrictions on self-persuasion costs and their across-consumer variation. First, it constrains the overall pattern of MSPCs to be hyperbolic. Second, the forms proposed in (4), together with $\sigma_0, \sigma_1 \geq 0$, ensure a self-persuasion map of non-crossing nested contours similar to a well-behaved indifference map.[5] This means an individual who initially prefers a product more strongly than another individual finds it less costly to achieve any given level of attitude toward that product than the other individual. This in turn implies consumers will never "leapfrog" other consumers' preference positions through self-persuasion: to assume otherwise would admit a level of ineptitude at self-persuasion by individuals with strong initial preferences for which there is no conceptual justification.

Beyond these, I impose three additional restrictions. The requirement that advertising improves consumers' self-persuasion technology monotonically, $g^{j,0}(i,x) > g^{j,1}(i,x)$, translates directly under my parameterization into

Assumption 1. (Advertising-driven dominance). $\theta > \sigma_0 - \sigma_1$.

Second, I restrict self-persuasion costs to be positive both before and after advertising. This restriction takes the form of size limits on σ_0 and $\sigma_1 + \theta$ relative to b.

Assumption 2. $\sigma_0 < b$ *and* $\sigma_1 + \theta < b$.

Third, the pre- and post-advertising levels of self-persuasion mean-regressivity - σ_0 and σ_1, respectively - must be constrained relative to the size of the corresponding marginal net benefit to self-persuasion of the marginal consumer - $t - b$ and $t - b + \theta$, respectively. These are, in effect, restrictions that the size of ν imposes on σ_0 and σ_1, mediated by a calibrating constant r. The restrictions ensure a unique equilibrium in finite prices. The precise size of r characteristic to the assumption is fixed to satisfy Lemma 3, which appears in the proof of the equilibrium existence result set out in the appendix.

Assumption 3. $\sigma_0 \leq \frac{r\nu}{\frac{1}{2}-\nu}(t-b)$ *and* $\sigma_1 \leq \frac{r\nu}{\frac{1}{2}-\nu}(t-b+\theta)$ *for* r *satisfying Lemma 3.*

It should be noted that Assumption 3 does not limit the possible values of σ_0 and σ_1 per se, but rather restricts them relative to corresponding values of ν. It is straightforward to see that for any given r, all values $\sigma_0, \sigma_1 > 0$ are in principle plausible under

[5] Formally, for all $x \in [\nu, 1-\nu]$, $-\frac{\partial g^0}{\partial x} < \frac{\partial g^0}{\partial i}$ (and $\frac{\partial g^1}{\partial x} < \frac{\partial g^1}{\partial i}$).

Assumption 3 given ν sufficiently close to $\frac{1}{2}$. Intuitively, a highly centralized distribution of consumers allows for a large relative self-persuasion cost advantage for inframarginal consumers (i.e., low mean-regressivity) without destabilizing the equilibrium in prices, as incentives to raise price are kept in check by a high concentration of consumers at the margin. (The general intuition of the relationship of the mean-regressivity of self-persuasion to prices will be discussed in Section 4.)

That the range of values possible for σ_0 and σ_1 is not bounded above as ν approaches $\frac{1}{2}$ is confirmed by the following lemma, which follows trivially from Lemma 3:

Lemma 1. *There exists $\bar{r}(\nu) > 0$ satisfying Lemma 3 as ν approaches $\frac{1}{2}$.*

Self-persuasion capital and market shares

Given MSPCs, a consumer's decision of whether to self-persuade with respect to a presumptive chosen product is taken at the margin by comparing the cost of incremental self-persuasion with the transportation cost that the consumer would incur in the event of not moving closer to the product. Where some self-persuasion is preferred relative to leaving attitude unimproved, the consumer's *self-persuasion capital* is defined as how much total attitude improvement he will attain given his preferences, his particular self-persuasion capabilities/costs, and the transportation cost (i.e., his opportunity cost of self-persuasion). Increasing and convex MSPCs and constant transportation costs imply uniquely defined self-persuasion capital. Define the set $X_j(t, A_j) := \{x \colon g^j(0, x, A_j) \leq t\}$. One may then define the implicit function on $i^{*j}(x, t, A_j) : X_j(t, A_j) \times \{t > 0\} \times \{A_j \geq 0\} \to \mathbb{R}^+$ such that $g^j(i^{*j}(x, t, A_j), x, A_j) = t$ as consumer x's "self-persuasion capital given t." Note that for $x \notin X_j(t, A_j)$, $i^{*j}(x, t, A_j) = 0$. The self-persuasion model thus nests non-self-persuasion as a sub-case (i.e., $X_j(t, A_j) = \emptyset$ for relevant A_j).

As it may be verified using (4) that $\frac{\partial g^j}{\partial i^{*j}}$ is nonzero on the full domain of g, one may solve explicitly for the functions i^{*j} on domain $X_j(t, A_j) \times \{t > 0\} \times \{A_j > 0\}$ (henceforth, I suppress the argument t):

$$\begin{aligned} i^{*0}(x, A_0) &= x - \frac{bx}{t + \phi(A_0)[(1-2x)\sigma_1 + \theta] + [1 - \phi(A_0)](1-2x)\sigma_0} \\ i^{*1}(x, A_1) &= 1 - x - \frac{b(1-x)}{t + \phi(A_1)[(2x-1)\sigma_1 + \theta] + [1 - \phi(A_1)](2x-1)\sigma_0} \end{aligned} \quad (5)$$

As can be seen from these expressions, the model has been calibrated so that $b \geq t$

corresponds, without advertising, to no self-persuasion by any consumer when $\sigma_0 = \sigma_1 = 0$ and no self-persuasion by consumers at a distance greater than or equal to $\frac{1}{2}$ from the product regardless of the values of σ_0 and σ_1. Using self-persuasion by the consumer at $x = \frac{1}{2}$ as a benchmark, the size of b relative to t when $b < t$ is an effective index of the self-persuasion consumers do in the absence of advertising: the smaller the ratio, the more self-persuasion.

As the model's purpose is to analyze the effect of advertising in influencing a self-persuasion process in which consumers are already naturally engaged, I restrict attention to the case in which all consumers do some self-persuasion in the absence of advertising - that is, $X_j(t, 0) = [0, 1]$ for $j = 0, 1$:

Assumption 4. $b < t$.

Accounting for self-persuasion, the utility of a consumer at x buying product 0 is given in general form by

$$U_0 = V - p_0 - t\left[x - i^{*0}(x, A_0)\right] - \int_0^{i^{*0}(x,A_0)} g^0(i, x, A_0)\, di \tag{6}$$

and, for a consumer at x buying product 1, by

$$U_1 = V - p_1 - t\left[1 - x - i^{*1}(x, A_1)\right] - \int_0^{i^{*1}(x,A_1)} g^1(i, x, A_1)\, di \tag{7}$$

Utility losses accruing to choosing a non-ideal product equal the sum of self-persuasion cost and transportation cost components and are a function of the consumer's self-persuasion capital. Figure 4 displays these losses graphically as areas under the MSPC and transportation cost curve.[6]

Following Nagler (2021), I impose what is in effect a restriction that V be sufficiently large relative to t:

Assumption 5. $\left\{V - t\left[x - i^{*0}(x)\right] - \int_0^{i^{*0}(x)} g^0(i, x)\, di\right\} \frac{x-\nu}{1-2\nu}$ *is increasing for all* $x \in [0, 1]$.

[6] Note that the setup in (6) and (7) is isomorphic to a traditional Hotelling model with nonlinear transportation costs. The rationale for layering self-persuasion into the model explicitly (i.e., by means of the "self-persuasion map") is so that its effects may be seen distinctly.

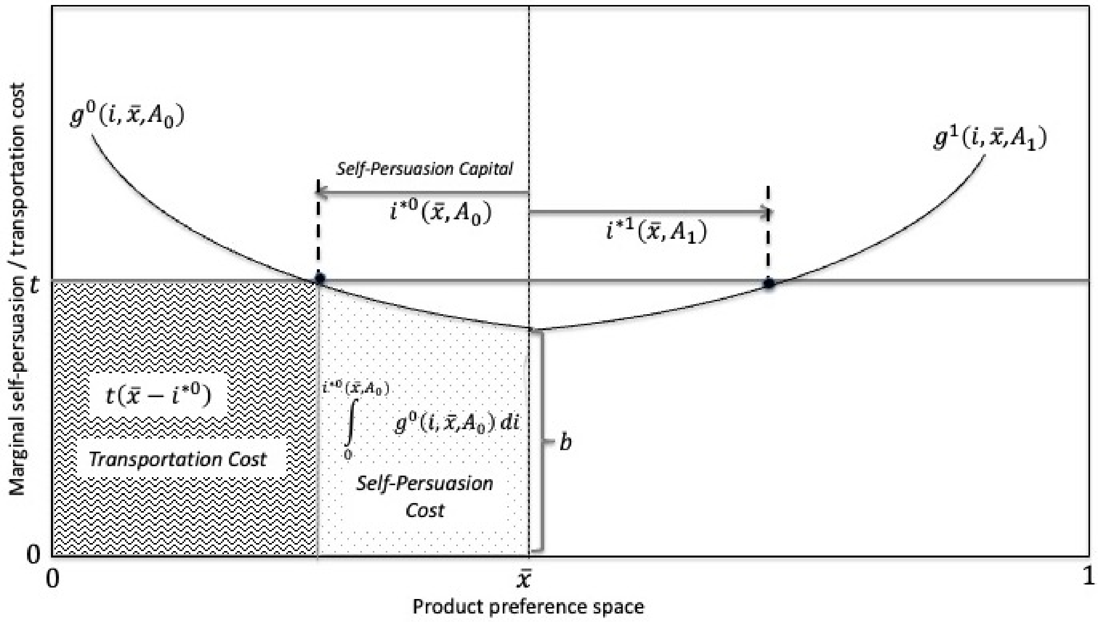

Figure 4: Components of Utility Loss from Selecting Non-ideal Product 0

The assumption is a sufficient condition for the market to be covered under self-persuasion:

Lemma 2. *Given Assumption 5, the market is covered in equilibrium.*

Lemma 2 ensures, in the absence of a corner solution, that there exists a unique location occupied by an indifferent consumer. This location, x_I, may be derived by setting $U_0 = U_1$, thus it is defined implicitly for general g^j and i^{*j} by

$$\begin{aligned}\Theta\left(x_I,t,p_0,p_1,A_0,A_1\right) \equiv\ & p_1 - p_0 + t - 2tx_I - t\left[i^{*1}\left(x_I,A_1\right) - i^{*0}\left(x_I,A_0\right)\right] \\ & + \int_0^{i^{*1}(x_I,A_1)} g^1\left(i,x_I,A_1\right)di - \int_0^{i^{*0}(x_I,A_0)} g^0\left(i,x_I,A_0\right)di = 0 \end{aligned} \tag{8}$$

Market shares for the two products are then defined by $D_0 = \frac{x-\nu}{1-2\nu}$ and $D_1 = 1 - \frac{x-\nu}{1-2\nu}$.

Advertising-pricing game

There are two periods. In $t = 1$, firms choose advertising expenditures, taking each other's advertising expenditure choices as given. In $t = 2$, they choose prices, taking each other's prices and their previous advertising choices as given. This ordering is consistent with much of the persuasive advertising literature (e.g., Bloch and Manceau 1999, Tremblay and Polasky 2002, Chioveanu 2008, Baye and Morgan 2009) in which advertising is perceived in effect as creating a long-term commitment to a particular pricing strategy[7] and contrasts with purely informative advertising which is broadly seen as being instantly changeable and useful inter alia for communicating pre-determined prices.[8] Logistically it reflects the recognition that a persuasion strategy is developed in parallel with product positioning, which in turn forms the basis for pricing.[9]

Firms recognize that their prices and their rivals' prices will depend on their prior advertising choices and so treat these strategically with respect to their advertising decisions in $t = 1$. At the end of $t = 2$, consumers choose products and contemporaneously adjust to the product they choose, accounting for the effects of their adjustment when making their choice. The consumers receive utility, and the firms earn profits. I seek subgame perfect Nash equilibria to this game.

Given demand, profits of the firms are given by

$$\begin{aligned} \Pi_0 &= p_0\left(A_0, A_1\right) \frac{x-\nu}{1-2\nu} - aA_0 \\ \Pi_1 &= p_1\left(A_0, A_1\right) \left(1 - \frac{x-\nu}{1-2\nu}\right) - aA_1 \end{aligned} \tag{9}$$

where a is the unit cost of advertising.

4 Equilibrium

Equilibrium in the two-stage game is solved by backward induction: one first must determine the Nash equilibrium price-setting strategies of the firms in $t = 2$, then the $t = 1$ advertising strategies that take account of the $t = 2$ decisions. The uniqueness of the equilibrium with respect to prices follows from the existing setup; only the concavity of ϕ is necessary as an additional condition to ensure the uniqueness of each firm's optimizing advertising level. I exploit additionally the symmetry of the setup.

Proposition 1. *If ϕ is sufficiently concave, then there exists a unique pure strategy Nash*

[7] See, in particular, von der Fehr and Stevik (1998)

[8] See, for example, Tremblay and Polasky (2002), p256; and Hefti and Liu (2020), p407.

[9] See, for example, https://www.linkedin.com/pulse/repositioning-product-helps-2x-its-price-sumit-jain?trk=public_profile_article_view.

equilibrium in which

$$p_0^* = p_1^* = (1-2\nu)\left\{-\phi(A_0)\theta + b - b\ln\left(\frac{b}{t+\phi(A_0)\theta}\right) + [\sigma_0 + (\sigma_1-\sigma_0)\phi(A_0)]\left[1-\frac{b}{t+\phi(A_0)\theta}\right]\right\},$$

$A_0^* = A_1^* \geq 0$ *and* $x_I^* = \frac{1}{2}$.

The price effect of advertising follows simply by differentiating the expression for price in Proposition 1 with respect to own advertising level:

Proposition 2. *Advertising increases prices if*

$$(\sigma_1-\sigma_0)\cdot(t-b) + \left(\sigma_0 - \frac{t^2-bt}{b}\right)\cdot\frac{b}{t}\theta > 0 \quad (10)$$

and decreases them if the inequality is reversed.

Proposition 2 shows that advertising affects prices via the weighted sum of two component effects: a *shaping effect* and a *manifestation effect*. The shaping effect, reflected in the first term of (10), arises from how advertising affects the relative mean-regressivity of the self-persuasion map. This component implies lower prices if advertising increases mean-regressivity by targeting marginal consumers ($\sigma_1 < \sigma_0$) and higher prices if advertising decreases mean-regressivity by targeting inframarginal consumers ($\sigma_1 > \sigma_0$). The manifestation effect, reflected in the second term, arises from advertising's further manifestation - through its mass-market effect θ - of self-persuasion's existing effect on price as reflected in the initial map - or, more precisely, the pre-advertising level of self-persuasion mean-regressivity σ_0. It implies a decrease in prices if the initial map is sufficiently mean-regressive (that is, if $\sigma_0 < \frac{t^2-bt}{b}$), an increase in prices otherwise ($\sigma_0 > \frac{t^2-bt}{b}$).[10]

To understand the price effects inherent in these components, it is necessary to appreciate why the relative mean-regressivity of self-persuasion is conducive to higher price sensitivity, hence lower prices. Consider again the two panels of Figure 3. In the relatively mean-regressive map shown in the first panel, the curves start at the same baseline and grow progressively steeper as one moves from $x_I^* = 0.5$ toward positions of stronger initial preference. Under this regime, a price increase for one of the products moves to the margin previously-inframarginal consumers who find self-persuasion less productive

[10]Recall that, in view of Lemma 1, Assumption 3 does not impose an absolute upper bound on σ_0, so all such values are possible in equilibrium.

at improving their attitude than the consumer at x_I^*. These consumers, if they switched products, would forgo higher total costs (i.e., transportation plus self-persuasion costs) from the product they left than would the consumer at x_I^*. Given symmetry, they would also incur lower total costs from their new product relative to the consumer at x_I^*. Such a pattern implies greater consumer price sensitivity relative to a market consisting of consumers displaying a less mean-regressive self-persuasion pattern; thus advertising shaping the map in this direction implies lower prices. Meanwhile, advertising that intensifies self-persuasion uniformly across consumers would further manifest the effect borne out by this particular map, increasing consumer price sensitivity and causing prices to fall, all else equal.

The curves shown in the second panel of Figure 3 rise just as steeply as those in the first panel, but - moving from $x_I^* = 0.5$ toward positions of stronger initial preference - they begin at progressively lower baseline values. Under this regime, a price increase for one of the products moves to the margin previously-inframarginal consumers who find self-persuasion relatively more productive at improving their attitude as compared to analogous consumers in the first panel. If they switched products, such consumers would forgo lower total costs from the product they left relative to consumers under the more mean-regressive regime shown in the first panel and, given symmetry, incur *higher* total costs from their new product relative to consumers reflected in the first panel. Such a pattern implies lower consumer price sensitivity. Advertising that reshapes self-persuasion to be less mean-regressive therefore raises prices, all else equal. And advertising that intensifies self-persuasion uniformly across consumers would further manifest the effect borne out by this particular map, one relatively less conducive to falling prices than the more mean-regressive map and perhaps conducive to increased prices.

As (10) makes evident, whether the shaping effect or manifestation effect dominates the overall price effect depends upon the relative size of $\sigma_1 - \sigma_0$ and $\sigma_0 - \frac{t^2-bt}{b}$, as well as the size of multiplicative weighting factors, $t-b$ and $\frac{b}{t}$, respectively. Those weights reflect the relative amount of self-persuasion that consumers do in the absence of advertising. Intuitively, a greater amount of such pre-existing self-persuasion (i.e., b small relative to t) implies more sizable pre-existing effects due to self-persuasion which are there for the advertising, in turn, to "shape." Conversely, the manifestation effect dominates the shaping effect when b is large relative to t, that is, when pre-advertising self-persuasion is relatively unimportant as compared with the self-persuasion engendered *through* the advertising.

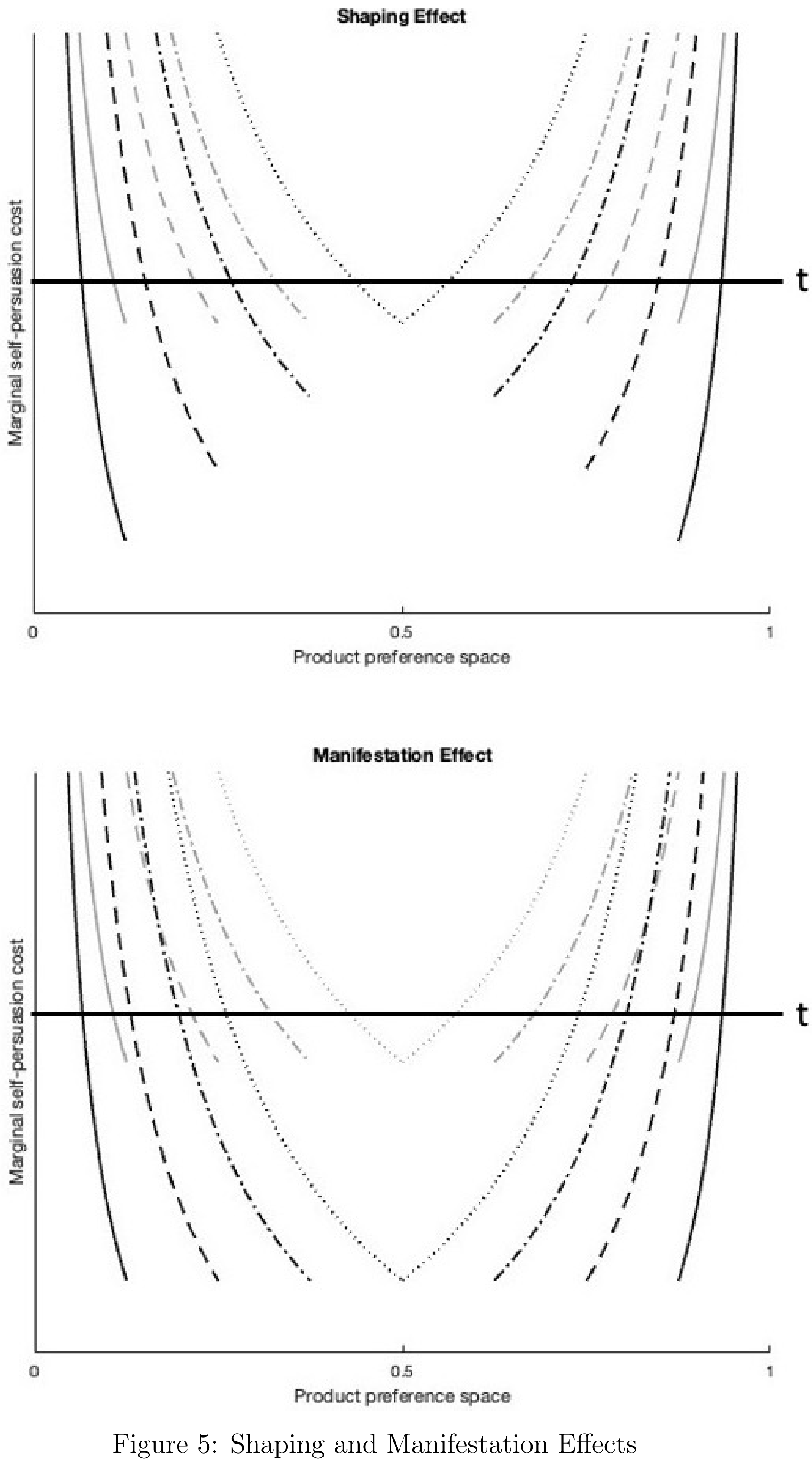

Figure 5: Shaping and Manifestation Effects

The relative importance of self-persuasion in the absence of advertising also influences the size of the manifestation effect itself. When b is small relative to t, such that $t-b$ and $\frac{t}{b}$ are large, $\frac{t^2-bt}{b}$ is larger, whereby a proportionally larger σ_0 is needed to achieve the same manifestation effect. Put in simple terms, the more the self-persuasion map is expressed (i.e., below t) pre-advertising, the less the inframarginal consumer advantage σ_0 matters to the relative costs of self-persuasion borne by different consumers. In essence, b small relative to t implies that the baseline mean-regressive pattern "washes out" σ_0, such that in turn that baseline pattern poses the dominant influence on price.

Figure 5 illustrates the component effects from Proposition 2 in the case where $\sigma_0 = 0$ and $\sigma_1 > 0$. The first panel illustrates the shaping effect: a vertical displacement in the $x \neq \frac{1}{2}$ self-persuasion curves proportional to their distance from $x = \frac{1}{2}$. The displayed example shows movement from the $\sigma_0 = 0$ pattern (grey) to the less mean-regressive pattern associated with $\sigma_1 > 0$ (black); this would imply higher prices, all else equal. The second panel illustrates the manifestation effect: an equal downward shift in *all* self-persuasion curves that maintains the shape of the *initial* mean-regressive map. The displayed example shows the $\sigma_0 = 0$ map becoming more fully expressed (i.e., extending further below t); this would imply lower prices, all else equal. The combined result could be higher or lower prices: which outcome prevails depends upon the relative sizes of $\sigma_1 - \sigma_0$ and $\left(\sigma_0 - \frac{t^2-bt}{b}\right)\theta$ and the sizes of the respective weights, $t-b$ and $\frac{b}{t}$.

Proposition 2's condition signing the price effect of advertising makes it possible to articulate a meaningful condition under which advertising will occur in strictly positive quantity in equilibrium:

Proposition 3. *If (i)* $-\theta(t-b) < \left[(\sigma_1 - \sigma_0)\cdot(t-b) + \left(\sigma_0 - \frac{t^2-bt}{b}\right)\cdot\frac{b\theta}{t}\right]$*, and (ii) the initial marginal productivity of advertising* $\phi'(A_j)\big|_{A_j=0}$ *is sufficiently large* ($j = 0, 1$)*, then the unique pure strategy Nash equilibrium consists of strictly positive amounts of advertising by both firms.*

The proposition is built on the logic that the firm will choose a positive quantity of advertising in equilibrium if advertising has a positive marginal revenue product *and* the first unit of advertising is sufficiently productive that the revenue produced by that unit is greater than its marginal cost. As condition (i) indicates, a positive effect of advertising on prices is a sufficient condition for its having a positive marginal revenue product, but not a necessary one. Advertising can decrease prices, so long as that effect is not large in size and is outweighed by the positive incremental impact of advertising on revenue through the firm's sales. That latter impact is larger the more productive advertising is in reducing the self-persuasion cost of the marginal consumer; this in turn depends on how much advertising shifts the MSPC function downward for that consumer (i.e., θ) and how much the marginal consumer is self-persuading pre-advertising (indexed by $t-b$).

5 Discussion

An essential contribution of the self-persuasion model set forth in the preceeding sections is its recognition that advertising's price effects do not depend solely on which consumers are targeted by the advertising, hence which experience an elevated WTP as a consequence of it - what the model characterizes as the shaping effect. They depend additionally on the latent pattern of self-persuasion that advertising brings to fuller expression via the manifestation effect. If the self-persuasion model is correct as to how persuasive advertising actually functions, what then are the implications for empirical work which presumes solely that advertising's influence consists in increasing the WTP of targeted consumers?

Erdem et al. (2008) offer a case in point. They estimate demand systems using a conditional indirect utility function – doing so in a manner that enables them to determine the effect of advertising on the demand curve as a whole, rather than merely on demand elasticity or the price level. This permits them to determine whether advertising raises the WTP of marginal consumers or inframarginal consumers. The paper offers an important advance over the demand elasticity-shifting metaphysics of earlier advertising empirical work (e.g., Benham 1972, Maurizi 1972, Steiner 1973, Cady 1976, Lambin 1976, Wittink 1977, Vanhonacker 1989) in that it views the effect of advertising on demand holistically.

However, viewed through the lens of the self-persuasion framework, the WTP-focused model the authors estimate is seen to be a reduced form. Its measurements merge the shaping effects of targeted advertising with the manifestation effects of mass-market advertising. Consequently, it is impossible to back out the advertising strategy from the observed price effects and, conversely, to predict the effects of any given strategy.[11]

Let us consider a simplified illustration. The model Erdem et al. estimate may be

[11] Indeed, Erdem et al. (2008) acknowledge the predictive limitations of their work, which they call "fundamentally descriptive" (p143).

specified, with brand subscripts omitted, as:

$$U_i = \alpha_i + \gamma_P P_i + \gamma_A A_i + \gamma_{PA} P_i \cdot A_i + \psi Z_i + \varepsilon_i \tag{11}$$

where U_i is the indirect utility of the consumer conditional on choice of the brand, P_i is the price of the brand faced by the consumer, A_i is the consumer's exposure to advertisements for the brand, and Z_i represents all other explanatory variables impacting the consumer. For the purposes of our illustration, let $\tilde{U}$ represent the consumer's utility from her best outside option, assume all variables other than α_i and ε_i are homogeneous across brands, and ignore the Z_i term. Then the consumer prefers the brand in question if and only if

$$\alpha_i + \gamma_P P + \gamma_A A + \gamma_{PA} P \cdot A + \varepsilon_i > \tilde{U}$$

This implies the consumer's WTP is

$$P = \frac{\alpha_i + \gamma_A A + \varepsilon_i - \tilde{U}}{-(\gamma_P + \gamma_{PA} A)}, \quad \text{where} \quad -(\gamma_P + \gamma_{PA} A) > 0$$

Differentiating this expression with respect to A and starting from a position of no advertising, one obtains

$$\left.\frac{dP}{dA}\right|_{A=0} = -\frac{\gamma_A}{\gamma_P} + \gamma_{PA} \frac{\alpha_i + \varepsilon_i - \tilde{U}}{\gamma_P^2}$$

Here, $\gamma_{PA} < 0$ represents what Erdem et al. (2008) refer to as Becker and Murphy's (1993) "presumptive case," in which advertising lowers the WTP of inframarginal consumers who have sufficiently large and positive values of $\alpha_i + \varepsilon_i - \tilde{U}$ while increasing the WTP of consumers for whom $\alpha_i + \varepsilon_i - \tilde{U}$ is zero or negative. Such advertising, the authors argue, flattens the demand curve, resulting in a higher elasticity of demand and lower prices. Erdem et al. go on to apply the model in (11) to data for 18 brands over four product categories, whereby they identify 17 of 18 brands as corresponding to the $\gamma_{PA} < 0$ presumptive case and one brand (Heinz ketchup) as corresponding to $\gamma_{PA} > 0$.

The trouble lies in where the analysis proceeds from here. One cannot distinguish whether the $\gamma_{PA} < 0$ case corresponds to the targeting of marginal consumers as Erdem et al. speculate (p141). In the self-persuasion model, such a case could indeed follow from $\sigma_0 - \sigma_1 > 0$ with $\theta > \sigma_0 - \sigma_1 > 0$. However, it could follow alternatively from

$\sigma_1 - \sigma_0 \geq 0$ if σ_0 is small enough and θ large enough. That is, the demand curve might have rotated in response to advertising in the manner that Erdem et al. measure, despite the advertising having targeted all consumers equally, or even primarily inframarginal consumers. A result along these lines follows in the event that, in addition to inducing the shaping effects traditionally associated with targeting, the advertising brings to the fore a strongly mean-regressive latent pattern of self-persuasion. One might argue that the manifestation of such a pattern should be the presumptive case in the absence of evidence about which consumers were targeted.

Conversely, in light of the self-persuasion framework, it is impossible to predict on the basis of the Erdem et al. model whether targeting advertising predominantly toward marginal consumers will actually result in lower prices. Higher prices could result under the marginal targeting case $\theta > \sigma_0 - \sigma_1 > 0$ if $\sigma_0 > \frac{t^2 - bt}{b}$, such that the advertising is manifesting a *non*-mean-regressive latent pattern of self-persuasion.

In sum, the self-persuasion model points out the necessity of not presuming consumers' responses to advertising based on a certain targeting strategy in trying to make predictions about price effects. Models that are agnostic to the basis of advertising's influence on WTP and therefore presume such responses risk making errors in prediction and drawing erroneous conclusions about causal connections. The consumers' responses must be explicitly measured. I discuss this possibility in the concluding section.

6 Conclusion

This paper has offered a model of persuasive advertising as distinct from informative advertising and with its own efficient purpose. Its conception of advertising's role arises from a new theory of consumer behavior that recognizes the phenomenon of motivated preference. When rational consumers self-persuade, it becomes possible to conceive of how non-informative advertising might not only influence them but benefit them directly.

The self-persuasion framework both faithfully reflects the psychological evidence on behavior and improves prospects for accurately predicting how advertising affects prices. Perhaps the most consequential contribution of my approach is in identifying the potential for effects arising from latent patterns of consumer self-persuasion that advertising brings to fuller expression.

Future research efforts should be focused on developing new empirical approaches that utilize the self-persuasion framework to generate more precise and accurate predictions of advertising's effects. One possible approach would be to use advertising field experiments. Using scanner panels similar to Erdem et al. (2008) and measuring purchases, prices, ad exposures and relevant correlates within brand over time based on (11), one could implement an experimental design in which a randomly assigned treatment group is exposed to brand advertising, while remaining panel participants are not exposed. Post-treatment measurements would serve to identify, and thereby control for, the manifestation effect of advertising on price identified in Proposition 2. Such measurements could form the basis for predictions of price effects resulting from advertising campaigns targeted in different ways from the original experimental treatment. For example, advertising targeted more heavily toward marginal consumers would be predicted to result in lower prices *relative to the baseline* established by the measured treatment effect. Such predictions could be tested with subsequent treatments. The sensitivity of these subsequent treatment effects to the size and valence of manifestation effect components across brands could also be measured.

The approach presented in this paper has limitations. Certain features of persuasion have not be modeled in my simple framework. Price likely plays a role in persuading, rather than just being the result of persuasion. This might have distinct implications for prices and for the use of advertising, and it should be considered further. Additionally, the model has considered the effect of self-persuasion on its own and thus has not explicitly incorporated the effects on price of advertising with a both non-informative and informative components. The interaction of advertising's persuasive and informative roles deserve additional investigation.

A Appendix

Proof of Lemma 2

The proof is an extension of the proof of Bloch and Manceau's (1999) Lemma 1. Suppose that the market is not covered, that is, at equilibrium prices (p_0^*, p_1^*) there exists a consumer x for whom

$$V - p_0^* - t\left[x - i^{*0}(x)\right] - \int_0^{i^{*0}(x_0)} g^0(i, x)\, di < 0$$

and

$$V - p_1^* - t\left[1 - x - i^{*1}(x)\right] - \int_0^{i^{*1}(x_1)} g^1(i,x)\,di < 0$$

One can show these prices do *not* constitute a Nash equilibrium, in that firm 0 can increase its profit by lowering its price p_0 without altering the profit, hence strategy, of firm 1. Begin by noting that, under (p_0^*, p_1^*), because there is a consumer for whom neither good provides nonnegative utility somewhere between the firms, the profit of firm 0 can be written

$$\Pi_0 = p_0^*(x_0)\frac{x_0 - \nu}{1 - 2\nu} \equiv \left\{ V - t\left[x_0 - i^{*0}(x_0)\right] - \int_0^{i^{*0}(x_0)} g^0(i, x_0)\,di \right\} \frac{x_0 - \nu}{1 - 2\nu} \tag{A.1}$$

where x_0 is the position of the consumer who, at prices (p_0^*, p_1^*), is just indifferent between buying product 0 and buying nothing. By assumption, $\partial\Pi_0/\partial x_0 > 0$. Now note that

$$\frac{\partial p_0}{\partial x_0} = -t + t\frac{\partial i^{*0}}{\partial x} - g^0\left(i^{*0}, x_0\right)\frac{\partial i^{*0}}{\partial x} - \int_0^{i^{*0}(x_0)} \frac{\partial g^0}{\partial x_0} di = -t - \int_0^{i^{*0}(x_0)} \frac{\partial g^0}{\partial x_0} di \tag{A.2}$$

Using (4), we obtain

$$\frac{\partial g^0}{\partial x} = -\frac{bi}{(x-i)^2} + 2\sigma_1\phi(A_0) + 2\sigma_0\left[1 - \phi(A_0)\right] \tag{A.3}$$

and

$$\frac{\partial g^0}{\partial i} = \frac{bx}{(x-i)^2} > -\frac{\partial g^0}{\partial x_0}$$

thus it follows from (A.2) that

$$\frac{\partial p_0}{\partial x_0} < -t + \int_0^{i^{*0}(x_0)} \frac{\partial g^0}{\partial i} di = -t + g\left(i^{*0}(x_0), x_0\right) - g(0, x_0) = -g(0, x_0) < 0$$

Since $\partial\Pi_0/\partial x_0 = \left(\partial\Pi_0/\partial p_0\right)\left(\partial p_0/\partial x_0\right)$, it follows that $\partial\Pi_0/\partial p_0 < 0$. Therefore a small downward deviation in the price p_0 from p_0^* increases firm 0's profits while not affecting firm 1's profits. This contradicts the assertion that (p_0^*, p_1^*) constitutes an equilibrium.

Proof of Proposition 1

I begin by proving the uniqueness of the equilibrium price vector. For this purpose, and since advertising strategies are decided before price strategies, I will predicate on a unique and symmetric equilibrium advertising vector $A_0^* = A_1^*$ and define $\bar{\phi} \equiv \phi(A_0^*) = \phi(A_1^*)$. Let us further define

$$\begin{aligned} Y &\equiv t + \bar{\phi}\left[(2x_I - 1)\sigma_1 + \theta\right] + \left(1 - \bar{\phi}\right)(2x_I - 1)\sigma_0 \\ Z &\equiv t + \bar{\phi}\left[(1 - 2x_I)\sigma_1 + \theta\right] + \left(1 - \bar{\phi}\right)(1 - 2x_I)\sigma_0 \\ \Sigma_0 &\equiv 2x_I\left[\sigma_1\bar{\phi} + \sigma_0\left(1 - \bar{\phi}\right)\right] \\ \Sigma_1 &\equiv 2(1 - x_I)\left[\sigma_1\bar{\phi} + \sigma_0\left(1 - \bar{\phi}\right)\right] \\ K &\equiv (x_I - \nu)\left(\frac{2Y^2 - 2bY - b\Sigma_1}{(1 - x_I)Y^2}\Sigma_1 - \frac{2Z^2 - 2bZ - b\Sigma_0}{x_I Z^2}\Sigma_0\right) \end{aligned} \tag{A.4}$$

Clearly, $\Sigma_0 \geq 0$ and $\Sigma_1 \geq 0$, and Assumptions 2 and 4 ensure $Y > 0$ and $Z > 0$.

Proposition 1 is proved with two intermediate lemmas.

Lemma 3. *For any $\nu \in \left[0, \frac{1}{2}\right]$, there exists $r > 0$ such that $K < 2t - (Z - \Sigma_0)\frac{Z-b}{Z} - (Y - \Sigma_1)\frac{Y-b}{Y} - b\ln\frac{b}{Y} - b\ln\frac{b}{Z}$ for $\sigma_0 \leq \frac{r\nu}{\frac{1}{2}-\nu}(t - b)$, $\sigma_1 \leq \frac{r\nu}{\frac{1}{2}-\nu}(t - b + \theta)$.*

Proof. Given the symmetry of K, $x_I = 1-\nu$ is the value of $x_I \in [\nu, 1-\nu]$ that maximizes K. Setting $x_I = 1 - \nu$ and noting $\frac{\Sigma_0}{1-\nu} = \frac{\Sigma_1}{\nu} < \infty$ for all $\nu \in \left[0, \frac{1}{2}\right]$, K is seen to be bounded above on $\nu \in \left[0, \frac{1}{2}\right]$ and $\sigma_0, \sigma_1 \in (0, b)$, and zero-valued at $\sigma_0 = \sigma_1 = 0$. It is straightforward, moreover, to verify that $2t - (Z - \Sigma_0)\frac{Z-b}{Z} - (Y - \Sigma_1)\frac{Y-b}{Y} - b\ln\frac{b}{Y} - b\ln\frac{b}{Z} > 0$ at $\sigma_0 = \sigma_1 = 0$, and that both this expression and K are continuous in σ_0 and σ_1 on their domain $(0, b)$.

Set $\sigma_0 = \frac{t-b}{t-b+\theta}\sigma_1$. Subject to this restriction, it may be shown that K is monotone nondecreasing in σ_1: substituting $x_I = 1 - \nu$ into the expressions in (A.4) and differentiating yields

$$\begin{aligned} \frac{\partial Y}{\partial \sigma_1} &= (1 - 2\nu)\frac{t - b + \bar{\phi}\theta}{t - b + \theta} \geq 0 \\ \frac{\partial Z}{\partial \sigma_1} &= (2\nu - 1)\frac{t - b + \bar{\phi}\theta}{t - b + \theta} = -\frac{\partial Y}{\partial \sigma_1} \leq 0 \\ \frac{\partial \Sigma_1}{\partial \sigma_1} &= 2\nu\frac{t - b + \bar{\phi}\theta}{t - b + \theta} \geq 0 \\ \frac{\partial \Sigma_0}{\partial \sigma_1} &= 2(1 - \nu)\frac{t - b + \bar{\phi}\theta}{t - b + \theta} \geq \frac{\partial \Sigma_1}{\partial \sigma_1} \geq 0 \end{aligned}$$

whereby $\frac{\partial K}{\partial \sigma_1}$ reduces to

$$(1-2\nu)\left[\frac{\Sigma_1}{\nu}\frac{\partial Y}{\partial \sigma_1}\frac{2bYZ\left(Z^2+Y^2\right)+4b\left(Z^3\Sigma_1+Y^3\Sigma_0\right)}{Y^3Z^3}+\frac{b\left(Y-Z\right)}{YZ}\frac{2\frac{\partial \Sigma_0}{\partial \sigma_1}}{1-\nu}\right]\geq 0$$

I can now complete the proof. Fix an arbitrary $\tilde{\nu}\in\left[0,\frac{1}{2}\right)$. Given $K=0$ for $\sigma_0=\sigma_1=0$ and K continuous and monotone in σ_1, it is clear that there exists $\bar{\sigma}_{\nu=\tilde{\nu}}>0$ for which $K<2t-\left(Z-\Sigma_0\right)\frac{Z-b}{Z}-\left(Y-\Sigma_1\right)\frac{Y-b}{Y}-b\ln\frac{b}{Y}-b\ln\frac{b}{Z}$ is satisfied for all $\sigma_1\in\left[0,\bar{\sigma}_{\nu=\tilde{\nu}}\right]$ where $\sigma_0=\frac{t-b}{t-b+\theta}\sigma_1$. Now set $\sigma_0=\frac{r\nu}{\frac{1}{2}-\nu}\left(t-b\right)$ and $\sigma_1=\frac{r\nu}{\frac{1}{2}-\nu}\left(t-b+\theta\right)$. Under this new regime, it follows that there exists $r\left(\tilde{\nu}\right)>0$ for which $K<2t-\left(Z-\Sigma_0\right)\frac{Z-b}{Z}-\left(Y-\Sigma_1\right)\frac{Y-b}{Y}-b\ln\frac{b}{Y}-b\ln\frac{b}{Z}$ is satisfied for $\sigma_0\leq\frac{r(\tilde{\nu})\tilde{\nu}}{\frac{1}{2}-\tilde{\nu}}\left(t-b\right)$, $\sigma_1\leq\frac{r(\tilde{\nu})\tilde{\nu}}{\frac{1}{2}-\tilde{\nu}}\left(t-b+\theta\right)$. It remains to observe that the statement of the lemma holds trivially for $\nu=\frac{1}{2}$. The lemma follows. □

Lemma 4. *(Single Crossing Property)* $\frac{\frac{\partial x_I}{\partial p_1}}{x_I-\nu}$ *is decreasing in* p_0 *(and* $-\frac{\frac{\partial x_I}{\partial p_1}}{1-x_I-\nu}$ *is decreasing in* p_1*).*

Proof. Differentiation of $\frac{\frac{\partial x_I}{\partial p_1}}{x_I-\nu}$ yields the following sufficient condition:

$$\frac{\partial^2 x_I}{\partial p_0^2}\left(x_I-\nu\right)-\left(\frac{\partial x_I}{\partial p_0}\right)^2 \quad < \quad 0 \tag{A.5}$$

Applying Cramer's rule to (8) yields

$$\frac{\partial x_I}{\partial p_0}=-\frac{\partial x_I}{\partial p_1}=\frac{1}{\left[-2t+\int_0^{i^{*1}(x_I,A_1)}\frac{\partial g^1}{\partial x_I}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial g^0}{\partial x_I}di\right]}<0 \tag{A.6}$$

and differentiating yields

$$\begin{aligned}\frac{\partial^2 x_I}{\partial p_0^2}=&-\left[-2t+\int_0^{i^{*1}(x_I,A_1)}\frac{\partial g^1}{\partial x_I}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial g^0}{\partial x_I}di\right]^{-2}\cdot\\&\left[\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}\frac{\partial x_I}{\partial p_0}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}\frac{\partial x_I}{\partial p_0}di+\left.\frac{\partial g^1}{\partial x_I}\right|_{i^{*1}(x_I)}\cdot i_x^{*1}\frac{\partial x_I}{\partial p_0}-\left.\frac{\partial g^0}{\partial x_I}\right|_{i^{*0}(x_I)}\cdot i_x^{*0}\frac{\partial x_I}{\partial p_0}\right]\\=&-\left(\frac{\partial x_I}{\partial p_0}\right)^3\left[\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}di+\frac{\partial g^1}{\partial x_I}\left(i^{*1}\right)\cdot i_x^{*1}-\frac{\partial g^0}{\partial x_I}\left(i^{*0}\right)\cdot i_x^{*0}\right]\end{aligned} \tag{A.7}$$

where $\frac{\partial x_I}{\partial p_0}$, which is not a function of i, has been pulled out of the integrals; and where I represent $\left.\frac{\partial g^j}{\partial x_I}\right|_{i^{*j}(x_I)}$ in shorthand by $\frac{\partial g^j}{\partial x_I}\left(i^{*j}\right)$. Thus,

$$\frac{\partial^2 x_I}{\partial p_0^2}\left(x_I-\nu\right)-\left(\frac{\partial x_I}{\partial p_0}\right)^2 = -\left(\frac{\partial x_I}{\partial p_0}\right)^3\left(x_I-\nu\right)\left[\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}di+\frac{\partial g^1}{\partial x_I}\left(i^{*1}\right)\cdot i_x^{*1}-\frac{\partial g^0}{\partial x_I}\left(i^{*0}\right)\cdot i_x^{*0}\right]-\left(\frac{\partial x_I}{\partial p_0}\right)^2$$

which takes the sign of

$$-\frac{\partial x_I}{\partial p_0}\left(x_I-\nu\right)\left[\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}di+\frac{\partial g^1}{\partial x_I}\left(i^{*1}\right)\cdot i_x^{*1}-\frac{\partial g^0}{\partial x_I}\left(i^{*0}\right)\cdot i_x^{*0}\right]-1$$

Dividing by $-\frac{\partial x_I}{\partial p_0}$ and using (A.6), this takes the sign of

$$\left(x_I-\nu\right)\left[\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}di+\frac{\partial g^1}{\partial x_I}\left(i^{*1}\right)\cdot i_x^{*1}-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}di-\frac{\partial g^0}{\partial x_I}\left(i^{*0}\right)\cdot i_x^{*0}\right]+\int_0^{i^{*1}(x_I,A_1)}\frac{\partial g^1}{\partial x_I}di-\int_0^{i^{*0}(x_I,A_0)}\frac{\partial g^0}{\partial x_I}di-2t \tag{A.8}$$

The next step is to obtain the components of this expression. From (4) I derive

$$\frac{\partial g^0}{\partial x} = \left[-\frac{bi}{\left(x-i\right)^2}+2\sigma_1\phi\left(A_0\right)+2\sigma_0\left[1-\phi\left(A_0\right)\right]\right] \tag{A.9}$$

$$\frac{\partial g^1}{\partial x} = \frac{bi}{\left(1-x-i\right)^2}-2\sigma_1\phi\left(A_1\right)-2\sigma_0\left[1-\phi\left(A_1\right)\right] \tag{A.10}$$

$$\frac{\partial^2 g^0}{\partial x^2} = \frac{2bi}{\left(x-i\right)^3}$$

$$\frac{\partial^2 g^1}{\partial x^2} = \frac{2bi}{\left(1-x-i\right)^3}$$

This yields

$$\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}di = \frac{bi^{*1}}{\left(1-x_I-i^{*1}\right)^2}-\frac{b}{1-x_I-i^{*1}}+\frac{b}{1-x_I}$$

$$\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}di = \frac{bi^{*0}}{\left(x_I-i^{*0}\right)^2}-\frac{b}{x_I-i^{*0}}+\frac{b}{x_I}$$

Using the expressions in (5), these may be rewritten as

$$\int_0^{i^{*1}(x_I,A_1)}\frac{\partial^2 g^1}{\partial x_I^2}di = \frac{\left\{t-b+\phi\left(A_1\right)\left[\left(2x_I-1\right)\sigma_1+\theta\right]+\left[1-\phi\left(A_1\right)\right]\left(2x_I-1\right)\sigma_0\right\}^2}{b\left(1-x_I\right)} \tag{A.11}$$

$$\int_0^{i^{*0}(x_I,A_0)}\frac{\partial^2 g^0}{\partial x_I^2}di = \frac{\left\{t-b+\phi\left(A_0\right)\left[\left(1-2x_I\right)\sigma_1+\theta\right]+\left[1-\phi\left(A_0\right)\right]\left(1-2x_I\right)\sigma_0\right\}^2}{bx_I}$$

Meanwhile, using (A.9) and (A.10),

$$\int_0^{i^{*1}(x_I,A_1)} \frac{\partial g^1}{\partial x_I} di = \frac{bi^{*1}}{1-x_I-i^{*1}} + b\ln\left(\frac{1-x_I-i^{*1}}{1-x_I}\right) - 2\sigma_1\phi(A_1)\, i^{*1} - 2\sigma_0\left[1-\phi(A_1)\right] i^{*1} \quad \text{(A.12)}$$

$$\int_0^{i^{*0}(x_I,A_0)} \frac{\partial g^0}{\partial x_I} di = -\frac{bi^{*0}}{x_I-i^{*0}} - b\ln\left(\frac{x_I-i^{*0}}{x_I}\right) + 2\sigma_1\phi(A_0)\, i^{*0} + 2\sigma_0\left[1-\phi(A_0)\right] i^{*0} \quad \text{(A.13)}$$

And lastly, using (5),

$$i_x^{*0} = 1 - b\frac{t+\phi(A_0)\left[\sigma_1+\theta\right]+\left[1-\phi(A_0)\right]\sigma_0}{\left\{t+\phi(A_0)\left[(1-2x)\sigma_1+\theta\right]+\left[1-\phi(A_0)\right](1-2x)\sigma_0\right\}^2}$$

$$i_x^{*1} = -1 - b\frac{-t-\phi(A_1)(\sigma_1+\theta)-\left[1-\phi(A_1)\right]\sigma_0}{\left\{t+\phi(A_1)\left[(2x-1)\sigma_1+\theta\right]+\left[1-\phi(A_1)\right](2x-1)\sigma_0\right\}^2}$$

Using the definitions in (A.4) one may write

$$i_x^{*0}(x_I) = \frac{Z^2-b(Z+\Sigma_0)}{Z^2} \qquad i_x^{*1}(x_I) = \frac{-Y^2+b(Y+\Sigma_1)}{Y^2}$$

$$\frac{\partial g^0}{\partial x_I} = \frac{-Z^2+b(Z+\Sigma_0)}{bx_I} \qquad \frac{\partial g^1}{\partial x_I} = \frac{Y^2-b(Y+\Sigma_1)}{b(1-x_I)}$$

$$\int_0^{i^{*0}(x_I,A_0)} \frac{\partial^2 g^0}{\partial x_I^2} di = \frac{(Z-b)^2}{bx_I} \qquad \int_0^{i^{*1}(x_I,A_1)} \frac{\partial^2 g^1}{\partial x_I^2} di = \frac{(Y-b)^2}{b(1-x_I)}$$

and, using (5),

$$i^{*0} = x_I - \frac{bx_I}{Z} = \frac{x_I(Z-b)}{Z} \quad \text{(A.14)}$$

$$i^{*1} = 1-x_I-\frac{b(1-x_I)}{Y} = \frac{(1-x_I)(Y-b)}{Y} \quad \text{(A.15)}$$

Using (A.9) and (A.10),

$$\frac{\partial g^1}{\partial x_I} i_x^{*1} = -\frac{\left[Y^2-b(Y+\Sigma_1)\right]^2}{b(1-x_E^*)Y^2} \qquad \frac{\partial g^0}{\partial x_I} i_x^{*0} = -\frac{\left[Z^2-b(Z+\Sigma_0)\right]^2}{bx_I Z^2}$$

and, with some simplification,

$$\int_0^{i^{*1}(x_I,A_1)} \frac{\partial^2 g^1}{\partial x_I^2} di + \frac{\partial g^1}{\partial x_I} i_x^{*1} = \frac{2Y^2-2bY-b\Sigma_1}{(1-x_I)Y^2}\Sigma_1 \quad \text{(A.16)}$$

$$\int_0^{i^{*0}(x_I,A_0)} \frac{\partial^2 g^0}{\partial x_I^2} di + \frac{\partial g^0}{\partial x_I} i_x^{*0} = \frac{2Z^2-2bZ-b\Sigma_0}{x_I Z^2}\Sigma_0 \quad \text{(A.17)}$$

Additionally, using (A.12) and (A.13), combined with (A.14) and (A.15), I obtain

$$\int_0^{i^{*1}(x_I,A_1)} \frac{\partial g^1}{\partial x_I} di = (Y - \Sigma_1) \frac{Y-b}{Y} + b \ln \frac{b}{Y} \tag{A.18}$$

$$\int_0^{i^{*0}(x_I,A_0)} \frac{\partial g^0}{\partial x_I} di = -(Z - \Sigma_0) \frac{Z-b}{Z} - b \ln \frac{b}{Z} \tag{A.19}$$

Using (A.8) - with substitutions from (A.16), (A.17), (A.18), (A.19), and again (A.4) - the sufficient condition for $\frac{\frac{\partial x_I}{\partial p_1}}{x_I - \nu}$ decreasing in p_0 may be written as

$$\begin{aligned} (x_I - \nu) \left(\frac{2Y^2 - 2bY - b\Sigma_1}{(1-x_I) Y^2} \Sigma_1 - \frac{2Z^2 - 2bZ - b\Sigma_0}{x_I Z^2} \Sigma_0 \right) + (Y - \Sigma_1) \frac{Y-b}{Y} \quad + \\ (Z - \Sigma_0) \frac{Z-b}{Z} + b \ln \frac{b}{Y} + b \ln \frac{b}{Z} \quad < \quad 2t \qquad \text{(A.20)} \\ = K + (Y - \Sigma_1) \frac{Y-b}{Y} + (Z - \Sigma_0) \frac{Z-b}{Z} + b \ln \frac{b}{Y} + b \ln \frac{b}{Z} \quad < \quad 2t \end{aligned}$$

By Assumption 3 and Lemma 3, $K < 2t - (Z - \Sigma_0) \frac{Z-b}{Z} - (Y - \Sigma_1) \frac{Y-b}{Y} - b \ln \frac{b}{Y} - b \ln \frac{b}{Z}$, whereby the lemma follows. □

Now I turn to the Proposition itself. Differentiating firm 0's profit equation in (9) with respect to price yields

$$\frac{\partial \Pi_0}{\partial p_0} = \frac{x_I - \nu}{1 - 2\nu} + \frac{p_0}{1 - 2\nu} \frac{\partial x_I}{\partial p_0} \tag{A.21}$$

Using (A.21), one obtains $\left.\frac{\partial \Pi_0}{\partial p_0}\right|_{p_0=0} = \left.\frac{x_I - \nu}{1-2\nu}\right|_{p_0=0} > 0$: non-zero demand for product 0 is guaranteed at $p_0 = 0$ by Lemma 2 combined with symmetry. Moreover, $\left.\frac{\partial \Pi_0}{\partial p_0}\right|_{p_0|_{x_I=\nu}} = \frac{p_0}{1-2\nu} \frac{\partial x_I}{\partial p_0} < 0$, where $p_0|_{x_I=\nu} > 0$. Given this, it follows from Lemma 4 that there exists a unique solution to the first-order condition $\frac{\partial \Pi_0}{\partial p_0} = 0$ and it is $p_0^* = -\frac{x_I^* - \nu}{\frac{\partial x_I^*}{\partial p_0}}$. A corresponding analysis of firm 1's problem yields the unique solution to the first-order condition $\frac{\partial \Pi_1}{\partial p_1} = 0$ as $p_1^* = \frac{1 - x_I^* - \nu}{\frac{\partial x_I^*}{\partial p_1}}$. Symmetry yields $x_I^* = \frac{1}{2}$, whereby I have $p_0^* = p_1^* = -\frac{\frac{1}{2} - \nu}{\frac{\partial x_I^*}{\partial p_0}}$. Using (A.6) and the forms in (4), I obtain

$$p_0^* = p_1^* = (1 - 2\nu) \left\{ -\phi(A_0)\theta + b - b \ln \left(\frac{b}{t + \phi(A_0)\theta} \right) + [\sigma_0 + (\sigma_1 - \sigma_0)\phi(A_0)] \left[1 - \frac{b}{t + \phi(A_0)\theta} \right] \right\}$$

It remains to show that the uniqueness of the equilibrium advertising vector follows for ϕ sufficiently concave. Specifically, I endeavor to show that this condition is sufficient to establish that there exists $\bar{A}_0$ such that, for $A_0 > \bar{A}_0$, $\frac{\partial \Pi_0}{\partial A_0} < 0$; and that $\frac{\partial^2 \Pi_0}{\partial A_0^2} < 0$ everywhere. Differentiating firm 0's profit equation in (9) with respect to A_0 yields

$$\frac{\partial \Pi_0}{\partial A_0} = \frac{p_0}{1-2\nu}\left[\frac{\partial x_I}{\partial A_0} + \frac{\partial x_I}{\partial p_0}\frac{\partial p_0}{\partial A_0} + \frac{\partial x_I}{\partial p_1}\frac{\partial p_1}{\partial A_0}\right] + \frac{\partial p_0}{\partial A_0}\frac{x_I - \nu}{1-2\nu} - a \tag{A.22}$$

Firm 0 knows it will choose price optimally in $t = 2$ taking its advertising choice as given; accordingly firm 0's advertising decision involves substituting the first-order condition for price and the expression $p_0^* = -\frac{x_I - \nu}{\frac{\partial x_I}{\partial p_0}}$ into (A.22). Recognizing also that $\frac{\partial x_I}{\partial p_0} = -\frac{\partial x_I}{\partial p_1}$, (A.22) reduces to

$$\frac{\partial \Pi_0}{\partial A_0} = \frac{x_I - \nu}{1-2\nu}\left[-\frac{\partial x_I}{\partial A_0}\Big/\frac{\partial x_I}{\partial p_0} + \frac{\partial p_1}{\partial A_0}\right] - a \tag{A.23}$$

which is the sum of advertising's effect on revenue through the prices of both firms. (The direct effect of advertising on 0's sales through its own price drops out of the optimization due to the envelope theorem.)

An expression for $\frac{\partial p_1}{\partial A_j}$ may be derived by totally differentiating the first-order condition in price for firm 1 and applying Cramer's rule:

$$\frac{\partial p_1}{\partial A_j} = -\frac{\frac{\partial x_I}{\partial A_j} + p_1 \frac{\partial^2 x_I}{\partial p_1 \partial A_j}}{2\frac{\partial x_I}{\partial p_1} + p_1 \frac{\partial^2 x_I}{\partial p_1^2}} \tag{A.24}$$

Additionally, applying Cramer's rule to (8), I obtain, as a general form,

$$\frac{\partial x_I}{\partial A_0} = \frac{\partial x_I}{\partial p_0}\phi'(A_0) \int_0^{i^{*0}(x_I, A_0)} \left[g^{0,1} - g^{0,0}\right] di \tag{A.25}$$

Inspection of (A.24) and (A.25) reveals that the components of the marginal revenue product of advertising given in (A.23) that multiply $\phi'(A_0)$ are bounded above with respect to increases in $A_0 \geq 0$, implying that there exists $\bar{A}_0$ such that, for $A_0 > \bar{A}_0$, $\frac{\partial \Pi_0}{\partial A_0} < 0$. Because the marginal revenue product of advertising is a linear function of $\phi'(A_0)$, it will decline monotonely with A_0, if $\phi'(A_0)$ declines quickly enough with A_0. Thus, for $\phi(.)$ sufficiently concave, $\frac{\partial^2 \Pi_0}{\partial A_0^2} < 0$ is guaranteed, whence, given parallel results for firm 1, the existence of a unique equilibrium advertising vector. Symmetry of the problem and the restriction that advertising be nonnegative assure $A_0^* = A_1^* \geq 0$.

Proof of Proposition 3

Consider the firms' advertising choices at $t = 1$. I focus on analyzing firm 0's problem.

Sufficient conditions for $A_0^* > 0$ are conditions that can guarantee three outcomes: that $\frac{\partial \Pi_0}{\partial A_0} > 0$ at $A_0 = 0$; that there exists $\bar{A}_0$ such that, for $A_0 > \bar{A}_0$, $\frac{\partial \Pi_0}{\partial A_0} < 0$; and that $\frac{\partial^2 \Pi_0}{\partial A_0^2} < 0$ everywhere. The last two outcomes were established by Proposition 1, so here it is necessary only to establish what is required for the first outcome. It can be shown easily that the marginal revenue product of firm 0's advertising – the first term in (A.23) - is a linear function of $\phi'(A_0)$. Thus it is sufficient that the first unit of advertising be sufficiently productive (i.e., $\phi'(A_0)|_{A_0=0}$ sufficiently large, as per condition (i) of the proposition) and the marginal revenue product of advertising be positive at $A_0 = 0$. My task, then, is to establish the equivalence of (ii) to a positive marginal revenue product of advertising for firm 0 at $A_0 = 0$.

I use the marginal revenue product expression in (A.23). Using (A.24), and $\frac{\partial x_I}{\partial p_0} = -\frac{\partial x_I}{\partial p_1}$, it can be shown that

$$-\frac{\partial x_I}{\partial A_0}/\frac{\partial x_I}{\partial p_0} + \frac{\partial p_1}{\partial A_0} = -\frac{\left\{-\frac{\partial x_I}{\partial A_0} + p_1 \frac{\partial^2 x_I}{\partial p_1 \partial A_0} - \frac{\partial x_I}{\partial A_0} p_1 \left(\frac{\partial^2 x_I}{\partial p_1^2}/\frac{\partial x_I}{\partial p_1}\right)\right\}}{2\frac{\partial x_I}{\partial p_1} + p_1 \frac{\partial^2 x_I}{\partial p_1^2}} \tag{A.26}$$

Because the denominator is known to be positive by the second-order conditions for a maximum in price, signing this expression positive is equivalent to signing the curly-bracketed expression negative.

Recalling that $p_1^* = \frac{1-x_I}{\frac{\partial x_I}{\partial p_1}}$, the curly bracketed expression in (A.26) may be written

$$-\frac{\partial x_I}{\partial A_0} + \frac{(1-x_I)\frac{\partial^2 x_I}{\partial p_1 \partial A_0}}{\frac{\partial x_I}{\partial p_1}} - \frac{(1-x_I)\frac{\partial x_I}{\partial A_0}\frac{\partial^2 x_I}{\partial p_1^2}}{\left(\frac{\partial x_I}{\partial p_1}\right)^2} \tag{A.27}$$

Using (A.6),

$$\frac{\partial^2 x_I}{\partial p_1 \partial A_0} = \left[-2t + \int_0^{i^{*1}(x_I,A_1)} \frac{\partial g^1}{\partial x_I} di - \int_0^{i^{*0}(x_I,A_0)} \frac{\partial g^0}{\partial x_I} di\right]^{-2} \cdot$$
$$\left[\int_0^{i^{*1}(x_I,A_1)} \frac{\partial^2 g^1}{\partial x_I^2}\frac{\partial x_I}{\partial A_0} di - \int_0^{i^{*0}(x_I,A_0)} \left[\frac{\partial^2 g^0}{\partial x_I \partial A_0} + \frac{\partial^2 g^0}{\partial x_I^2}\frac{\partial x_I}{\partial A_0}\right] di + \frac{\partial g^1}{\partial x_I}\left(i^{*1}\right) \cdot i_x^{*1} \frac{\partial x_I}{\partial A_0} - \frac{\partial g^0}{\partial x_I}\left(i^{*0}\right)\left(i_{A_0}^{*0} + i_x^{*0}\frac{\partial x_I}{\partial A_0}\right)\right] \tag{A.28}$$

This and (A.7) allow $\frac{\partial^2 x_I}{\partial p_1 \partial A_0}$ to be related to $\frac{\partial^2 x_I}{\partial p_1^2}$ by the following expression:

$$\frac{\partial^2 x_I}{\partial p_1 \partial A_0} = \frac{\partial^2 x_I}{\partial p_1^2}\left(\tfrac{\partial x_I}{\partial A_0}/\tfrac{\partial x_I}{\partial p_1}\right) - \left[\int_0^{i^{*0}(x_I, A_0)} \frac{\partial^2 g^0}{\partial x_I \partial A_0} di + \frac{\partial g^0}{\partial x_I}\left(i^{*0}\right) \cdot i^{*0}_{A_0}\right]\left(\frac{\partial x_I}{\partial p_1}\right)^2 \tag{A.29}$$

Substituting into (A.27), I obtain

$$-\frac{\partial x_I}{\partial A_0} - (1 - x_I)\left[\int_0^{i^{*0}(x_I, A_0)} \frac{\partial^2 g^0}{\partial x_I \partial A_0} di + \frac{\partial g^0}{\partial x_I}\left(i^{*0}\right) \cdot i^{*0}_{A_0}\right]\frac{\partial x_I}{\partial p_1} \tag{A.30}$$

which yields a necessary condition for an interior solution to the advertising problem, evaluated at $A_0 = A_1 = 0$,

$$\tfrac{\partial x_I}{\partial A_0}/\tfrac{\partial x_I}{\partial p_1} > -(1 - x_I)\left[\int_0^{i^{*0}(x_I, A_0)} \frac{\partial^2 g^0}{\partial x_I \partial A_0} di + \frac{\partial g^0}{\partial x_I}\left(i^{*0}\right) \cdot i^{*0}_{A_0}\right] \tag{A.31}$$

Analogous analytics show that another necessary condition arises from firm 1's optimization, to wit (at $A_0 = A_1 = 0$),

$$\tfrac{\partial x_I}{\partial A_1}/\tfrac{\partial x_I}{\partial p_0} < x_I\left[\int_0^{i^{*1}(x_I, A_1)} \frac{\partial^2 g^1}{\partial x_I \partial A_1} di + \frac{\partial g^1}{\partial x_I}\left(i^{*1}\right) \cdot i^{*1}_{A_1}\right]$$

It is sufficient to work with (A.31). I evaluate using the forms in (4) and, following Proposition 1, apply the candidate symmetric equilibrium $x_I = \frac{1}{2}$. Using (A.25), $\frac{\partial x_I^*}{\partial p_0} = -\frac{\partial x_I^*}{\partial p_1}$, (4), and (5) obtains at $A_0 = 0$ and $x_I = \frac{1}{2}$,

$$\tfrac{\partial x_I}{\partial A_0}/\tfrac{\partial x_I}{\partial p_1} = \phi'(0)\,\theta i^{*0}\left(\frac{1}{2}, 0\right) = \frac{1}{2}\phi'(0)\,\theta\frac{t-b}{t} \tag{A.32}$$

Using (A.3), evaluate at $x_I = \frac{1}{2}$ and differentiate with respect to A_0, then evaluate at $A_0 = 0$, to obtain

$$\frac{\partial^2 g^0}{\partial x \partial A_0} = 2\sigma_1\phi'(0) - 2\sigma_0\phi'(0) = 2(\sigma_1 - \sigma_0)\,\phi'(0) \tag{A.33}$$

Similarly, evaluating (5) at $x_I = \frac{1}{2}$ and $A_0 = 0$ yields $i^{*0} = \frac{t-b}{2t}$, whereby I have

$$\int_0^{i^{*0}(x_I^*, A_0)} \frac{\partial^2 g^0}{\partial x_I^* \partial A_0} di = 2(\sigma_1 - \sigma_0)\,\phi'(0) \cdot \frac{t-b}{2t}$$

Again using (A.3) and evaluating at $x_I = \frac{1}{2}$, $A_0 = 0$, and $i^{*0} = \frac{t-b}{2t}$,

$$\frac{\partial g^0}{\partial x_I}\left(i^{*0}\right) = 2\left(\sigma_0 - \frac{t^2 - bt}{b}\right)$$

Using (5) and evaluating at $x_I^* = \frac{1}{2}$ yields $i^{*0}\left(x_I^*\right) = \frac{1}{2}\left[1 - \frac{b}{t+\phi(A_0)\theta}\right]$, whereby

$$i_{A_0}^{*0} = \frac{b}{2}\frac{\phi'\left(A_0\right)\theta}{\left[t + \phi\left(A_0\right)\theta\right]^2}$$

which, evaluated at $A_0 = 0$, yields $\frac{b\phi'(0)\theta}{2t^2}$. Thus, $\int_0^{i^{*0}\left(x_I^*, A_0\right)} \frac{\partial^2 g^0}{\partial x_I^* \partial A_0} di + \frac{\partial g^0}{\partial x_I^*}\left(i^{*0}\right) \cdot i_{A_0}^{*0}$ becomes

$$2\left(\sigma_1 - \sigma_0\right)\phi'\left(0\right) \cdot \frac{t-b}{2t} + 2\left(\sigma_0 - \frac{t^2 - bt}{b}\right) \cdot \frac{b\phi'\left(0\right)\theta}{2t^2} \tag{A.34}$$

Now, using (A.32) and (A.34), (A.31) can be written,

$$-\theta\left(t - b\right) < \left[\left(\sigma_1 - \sigma_0\right) \cdot \left(t - b\right) + \left(\sigma_0 - \frac{t^2 - bt}{b}\right) \cdot \frac{b\theta}{t}\right]$$

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