NEGATIVE EXTERNALITIES, NETWORK EFFECTS, AND COMPATIBILITY

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Positive network effects arise where incremental product use increases the utility of users of compatible products (user-positive effects), but also in situations where product use imposes negative externalities that selectively affect the adopters of incompatible alternatives (nonuser-negative effects). This paper compares the social optimality of firms' incentives for compatibility under these two regimes. Using a "location" model of differentiated products, I find that, under both regimes, incentives for unilateral action to increase compatibility tend to be suboptimal when firms' networks are close in size, but they may be excessive for small firms when networks differ greatly in size. The result is consistent with prior analysis of the user-positive context (e.g., Katz and Shapiro 1985), but challenges the intuition that activities involving negative externalities are always oversupplied in an unregulated market. Public policy implications are discussed.

Keywords: Networks; Network externalities; Differentiated products; Location models; Compatibility

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I. Introduction

There are many situations in which the value to a consumer from using a product increases, relative to alternatives, with the number of other users of the same product or compatible products. Such products are said to exhibit network effects. Almost exclusively, the network effects phenomena described in the literature involve positive consumption effects, or more precisely "user-positive effects," whereby each consumer that uses the product increases the utility of other users of the product or compatible products.[1] Sometimes these effects are direct, such as on a telecommunications network, where utility derives directly from the number of people one may contact using the network. Other times, the effects are indirect. For instance, electronic game platforms provide greater benefit to users when more gaming software is available for use on them, and this tends to occur if they have more users.

But network effects are not limited to users increasing each other's utility. They also occur for products, or on platforms, characterized by users imposing costs on other people while simultaneously enjoying some degree of insulation against those costs. Such "nonuser-negative effects" occur as a result of what one may call "selective" negative externalities. For example, sport utility vehicles (SUVs) impose greater risks of injury and death on other motorists than do cars, while at the same time providing their occupants with increased protection against these same risks relative to cars. Other examples include noisome products, ranging from cigarettes to noisy leaf blowers, for which adoption reduces the displeasure from others' use; and situations in which non-

[1] Such phenomena have been referred to as "positive consumption externalities" (e.g., Katz and Shapiro 1985, 1986; Economides 1996), but recent analysis calls into question whether the positive consumption effects that give rise to network effects are always truly externalities. See, e.g., Liebowitz and Margolis (1994) and Farrell and Klemperer (2007, p. 2020).

adopters of a product or platform, such as ISO certification or expensive interview suits, incur a stigma that increases with the number of adopters.[2] In all these examples, incremental adopters increase other agents' preferences for adopting the product or platform relative to its alternatives because they impose external costs *selectively* (i.e., exclusively or to a greater extent) on non-adopters. The result is what one might call an "if-you-can't-beat-'em-join-'em" (IYCBEJE) network effect, whereby consumers join a bandwagon created by the increased undesirability of the alternative choice (Nagler 2011).

A key feature of the performance of markets with user-positive network effects is the scope of the relevant "network," that is, the scope of the product set through which the benefits of compatibility flow. Can, for example, the products of different firms be used together? (Katz and Shapiro 1985). It also matters *how* compatible other products are – are they fully interoperable, or with some degree of limitation or impedence? (Cremer, Rey, and Tirole 2000). These dimensions of compatibility affect not just the benefits derived by the individual user of the product, but also the competitive outcomes (e.g., prices and outputs), profits, and social welfare derived. A number of theoretical papers have found that profit-maximization-based decisions on who to extend compatibility to, and how fully, do not generally lead to socially optimal outcomes (Katz and Shapiro 1985, 1986; Economides and Flyer 1998; Church and Gandal 2000; Malueg and Schwartz 2006; Casadesus-Masanell and Ruiz-Aliseda 2009).

In markets involving nonuser-negative effects, compatibility comes into play just as it does with user-positive effects. When a consumer considers whether to use a selective-externality-imposing product or some alternative, she typically considers how

[2] For a more extensive list of examples and discussion, see Nagler (2011).

well or badly the alternative will fare in terms of its relationship to the imposing product and how many units of the imposing product there are in use. For example, the prospective buyer of a car might wonder how well she will make out if she collides with an SUV, and how many SUVs she is likely to encounter on the road. The first question has to do with compatibility, and the second with the size of the relevant installed base.[3]

A key strategic question facing the manufacturer in this context is how large to make the selective negative externality. That is, how *incompatible* should the product be with competing products?[4] For example, the manufacturers of SUVs must consider how dangerous to make their vehicles to the occupants of cars. The question of the social optimality of firms' incentives for compatibility in this case seems to have a trivially obvious answer. Because incompatibility directly increases relative preference through the IYCBEJE bandwagon, intuition suggests that private incentives for incompatibility would always be excessive. Public policy, one expects, could unambiguously improve welfare by reducing incompatibility at the margin.

This paper compares incentives for compatibility under user-positive and nonuser-negative network effects regimes and looks at both relative to the social optimum. I analyze a "location" model of differentiated products. In this sense, the approach is similar to the analyses of network externalities offered by Farrell and Saloner

[3] In the field of highway safety analysis, the first question is recognized as a compatibility issue, with studies referring to the "crash test compatibility" of different vehicles. See Bradsher (2002).

[4] External costs can often be manipulated through product design. SUVs generally have high, stiff front ends, and this increases the damage done to vehicles with which they collide; these effects could be undone through various design changes (Bradsher 2002, Latin and Kasolas 2002). Cigarettes could be manufactured to give off more or less smoke from the lit end, and the amount of smoke and noise emitted by gasoline-powered outdoor equipment could similarly be altered by design. Meanwhile, the size of the external costs that consumers *perceive* might be manipulated using marketing messages. For instance, calling greater attention to how imposing a particular SUV is might convince consumers that it is more dangerous to other motorists. (See Bradsher [2002] for examples of intimidating SUV advertisements.) By advertising, "Don't be the last programmer in the market to get one," a purveyor of computer programming certifications might enlarge perceptions of the stigma imposed on non-adopters by incremental adoptions.

(1992) and Matutes and Regibeau (1988, 1992), and different from the homogenous products model of Katz and Shapiro (1985). I focus on incentives for unilateral action on compatibility (e.g., in the case of user-positive effects, developing an adapter), rather than joint action (e.g., developing a standard). I also restrict attention to a static environment (i.e., a single-period model). My findings for user-positive effects essentially replicate the results of Katz and Shapiro (1985) concerning the relationship of firm size to compatibility incentives. But my findings for nonuser-negative effects do not bear out the intuition about excessive incentives. Instead, I find incentives for incompatibility that follow closely, though not exactly, Katz and Shapiro's results relating optimality of firms' incentives to network size: whereas firms that are close in size tend to have socially excessive incentives for incompatibility, an imposing firm has insufficient incentives for incompatibility if its "network" (customer base) is relatively very small or very large.

The next section lays out the general model. Section 3 derives welfare results for the user-positive case. Section 4 derives welfare results for the nonuser-negative case. Section 5 offers a public policy discussion and concludes.

II. A Model of Differentiated Product Duopoly with Network Effects

Consider a market for two products, A and B, sold at prices p_A and p_B, respectively. Consumers are distributed uniformly on a unit segment based on their preferences for A versus B, with the total number of consumers normalized to 1. There are no outside goods: consumers choose whether to purchase A or B, and each consumer will choose at most one unit of one of the two products. I posit a general framework of network effects as given by the following utility functions, representing the utility that the consumer located at a point *j* ($1 \geq j \geq 0$) obtains from purchasing a unit of product A or B, respectively:

$$U_A(j) = v + \theta - t(1-j) + \sigma_A \lambda Q_A + \sigma_{BX} \lambda Q_B - p_A \quad (1)$$

$$U_B(j) = v - \theta - tj + \sigma_B \lambda Q_B + \sigma_{AX} \lambda Q_A - p_B \quad (2)$$

Here, *v* represents the demand for all products; θ, which may be positive or negative, parameterizes the demand for A relative to B; *t* represents the intensity of consumers' relative preferences for A or B ($t > 0$); Q_i is the number of consumers who purchase product *i* ($i = A, B$); λ parameterizes the overall size of the network effect ($\lambda \geq 0$); and σ_i sizes and signs an *own* component of the network effect ($\sigma_i \in [-1,1]$), while σ_{iX} similarly sizes and signs a *cross* component of the network effect ($\sigma_{iX} \in [-1,1]$). A consumer who chooses neither A nor B receives utility of zero. Each consumer makes the choice that maximizes her utility.

Now, consider two cases: (I) $\sigma_A = \sigma_B = 1$, $\lambda > 0$ are given, and firm A sets $\sigma_X \equiv \sigma_{AX} = \sigma_{BX} \in [0,1]$; and (II) $\sigma_{AX} = -1, \sigma_A = \sigma_B = \sigma_{BX} = 0$ are given, and firm A sets λ. The first case is the classic case of a user-positive network effect: incremental users of A and B provide a benefit, λ, to other users of the same product. The decision that firm A faces is whether, and to what extent, to include firm B's consumers in the network. Does firm A makes B's consumers fully compatible with its own consumers, or partially compatible, or not at all? Here I assume, in Katz and Shapiro's (1985) parlance, that the compatibility technology is an "adapter," hence A and B are compatible if A unilaterally decides to undertake the expense to make them compatible. Note that the decision to make firm B's consumers compatible also means that firm A's consumers are compatible with firm B's, so that B's consumers receive increased network benefits as well; that is, the benefits are mutual. Since my purpose is to examine whether the level of compatibility chosen by a firm of a given network size is too high or too low, I assume without loss of generality that only A makes the decision of whether to make the products compatible.

The second case involves a nonuser-negative effect: firm A considers the possibility of imposing a negative externality that only affects the users of product B. I will show that the effect of doing this is also to create a network externality: when $\lambda > 0$, the reservation price of users of A increases with the number of users of A, all else equal.[5] Obviously, A's decision to make B's users more *incompatible* with product A does not have a mutual effect: A's users are not reciprocally harmed by users of B. That is, B is made more incompatible with A, but A is not made more incompatible with B.

The relative private and social incentives for compatibility in this case might seem obvious. As discussed in the introduction, since the incompatibility decision involves a unilaterally imposed negative externality, the incompatibility incentives of firm A would seem always to be excessive, unlike in the case of user-positive effects. The model considers whether that expectation is correct.

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[5] Nagler (2011) examines a more general framework allowing $\sigma_A \in [-1,0]$, so that the degree of "selectivity" of the negative externality is a parameter in the analysis,. The extreme case $\sigma_A = -1$ represents a pure negative externality due to the use of product A, with no consequent network externality; while varying values of $\sigma_A \in (-1,0]$ varies both the degree of selectivity and size of the network externality.

III. Equilibrium with User-Positive Effects

Setting parameters to the values proposed for case (I) above, (1) and (2) become:

$$U_A(j) = v + \theta - t(1 - j) + \lambda Q_A + \sigma_X \lambda Q_B - p_A \quad (3)$$

$$U_B(j) = v - \theta - tj + \lambda Q_B + \sigma_X \lambda Q_A - p_B \quad (4)$$

Assume *v* is large enough that all consumers choose A or B at equilibrium prices, implying $Q_A = 1 - Q_B$.[6] Combining (3) and (4) reveals that the consumer at *j* prefers A over B if $2\theta - t(1-2j) + (1-\sigma_X)\lambda(Q_A - Q_B) + p_B > p_A$. Therefore

$$\Psi_j = 2\theta - t(1-2j) + (1-\sigma_X)\lambda(Q_A - Q_B) + p_B \quad (5)$$

may be viewed as the consumer's reservation price for A relative to B. It is interesting also to note that the relative quantity of A versus B matters more to the relative willingness-to-pay the less compatible the two products are.

Following Katz and Shapiro (1985), we assume that firm A incurs a fixed cost of compatibility, $C(\sigma_X \lambda)$. This is assumed to depend upon the size of the compatibility benefit received by its users from each incremental user of B, and which B's users receive in turn from the incremental user of A. For simplicity, assume $C(\sigma_X \lambda) = k\sigma_X \lambda$ for $k > 0$. Firm A sets p_A and σ_X to maximize

$$\Pi_A = p_A Q_A - k\sigma_X \lambda \quad (6)$$

while firm B sets p_B to maximize

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[6] Consider $\overline{v} \equiv \theta + t + \varepsilon$ for arbitrary $\varepsilon > 0$. Then, for all $\lambda, \sigma_X > 0$, there exists $p_B > 0$ such that $U_B > 0$. Thus $\overline{v}$ satisfies the requirement.

$$\Pi_B = p_B Q_B \tag{7}$$

I restrict attention to $\lambda < t$, which is required for a stable interior solution; otherwise a small exogenous shift of consumers between products results, through the network effect, in all consumers shifting.

For an interior solution, $\Psi_{j^*} = p_A$, where j^* represents the threshold consumer (i.e., $Q_A = 1 - j^*$). Assuming such a solution, using (5), and making appropriate substitutions,

$$Q_A = \frac{t - p_A + p_B - (1-\sigma_X)\lambda + 2\theta}{2\left[t - (1-\sigma_X)\lambda\right]} \tag{8}$$

and

$$Q_B = 1 - Q_A = \frac{t - (1-\sigma_X)\lambda + p_A - p_B - 2\theta}{2\left[t - (1-\sigma_X)\lambda\right]} \tag{9}$$

The first-order conditions for firm A's profit maximization with respect to p_A and σ_X, respectively, are given by

$$\frac{t - (1-\sigma_X^*)\lambda - 2p_A^* + p_B^* + 2\theta}{2\left[t - (1-\sigma_X^*)\lambda\right]} = 0 \tag{10}$$

$$\frac{p_A^*\left(\frac{1}{2} - Q_A\right)}{\left[t - (1-\sigma_X^*)\lambda\right]} = k \tag{11}$$

while the first-order condition for firm B's problem is

$$\frac{t - (1-\sigma_X^*)\lambda + p_A^* - 2p_B^* - 2\theta}{2\left[t - (1-\sigma_X^*)\lambda\right]} = 0 \tag{12}$$

It is immediately clear from (11) that a corner solution is the only equilibrium when $Q_A \geq \frac{1}{2}$: firm A would like to set $\sigma_X < 0$ because the marginal benefit of compatibility at any positive level of compatibility is negative when firm A has more than half the market. Meanwhile, for $Q_A < \frac{1}{2}$, the smaller Q_A, the greater firm A wishes to set σ_X. Thus, the smaller the market share of a firm on this range, the greater its incentives for compatibility.

Solving (10) and (12) together yields

$$p_A^* = t - \left(1 - \sigma_X^*\right)\lambda + \tfrac{2}{3}\theta \tag{13}$$

and

$$p_B^* = t - \left(1 - \sigma_X^*\right)\lambda - \tfrac{2}{3}\theta \tag{14}$$

Hence, consistent with Farrell and Saloner (1992), compatibility implies higher prices. Incentives to cut price to achieve greater sales through enlargement of the own-product-specific network effect are diminished the more compatible the products are. Substituting (13) and (14) into (8) provides a useful partial-reduced-form for Q_A,

$$Q_A = \frac{t + \frac{2}{3}\theta - \left(1 - \sigma_X^*\right)\lambda}{2\left[t - \left(1 - \sigma_X^*\right)\lambda\right]} \tag{15}$$

Solving (11) explicitly for $\left(1 - \sigma_X^*\right)\lambda$ yields two roots:

$$\left(1 - \sigma_X^*\right)\lambda = t + \tfrac{1}{6k}\theta\left[1 \pm \sqrt{1 - 8k}\right] \tag{16}$$

As we demonstrate in the appendix, the values of σ_X^* that correspond to both roots are maxima. It is not necessary to our welfare results to determine which value of σ_X^* is preferred by firm A; we are able to proceed with (16). Substituting (16) into (13) and (14) yields the following corresponding equilibrium prices and quantities:

$$\left(p_A^*, p_B^*\right) = \left(\tfrac{2}{3}\theta - \tfrac{1}{6k}\theta\left[1 \pm \sqrt{1 - 8k}\right], -\tfrac{2}{3}\theta - \tfrac{1}{6k}\theta\left[1 \pm \sqrt{1 - 8k}\right]\right) \tag{17}$$

$$\left(Q_A^*, Q_B^*\right)=\left(\tfrac{1}{2}-\frac{2k}{1\pm\sqrt{1-8k}}, \tfrac{1}{2}+\frac{2k}{1\pm\sqrt{1-8k}}\right) \tag{18}$$

Thus, in my simple model, firm A uses compatibility over the range of an interior solution as a "buffer" to keep Q_A at an optimizing level that is independent of θ. A lower level of demand will cause A to set σ_X and p_A higher (hence, p_B will be higher as well – recall that prices rise with compatibility), keeping Q_A steady at the level given in (18). Meanwhile, when demand is high enough or low enough to correspond to a corner solution with respect to compatibility, firm A does not buffer its output. Equation (16) shows that $\sigma_X > 0$ requires $\theta < \frac{6k(\lambda - t)}{1\pm\sqrt{1-8k}} < 0$. At higher levels of demand, as inspection of (15) indicates, firm A sets $\sigma_X = 0$ and allows Q_A to vary positively with p_A. Meanwhile, $\sigma_X < 1$ requires $\theta > \frac{-6kt}{1\pm\sqrt{1-8k}}$.[7] When demand is below this lower threshold, firm A favors full compatibility, sets $\sigma_X = 1$, and again allows Q_A to vary positively with p_A.

We now turn to the question of how the level of compatibility chosen by firm A relates to the social optimum. Define welfare as

$$W \equiv \Pi_A + \Pi_B + \int_{j^*}^{1} U_A(j)\,dj + \int_{0}^{j^*} U_B(j)\,dj \tag{19}$$

Making substitutions from the model and differentiating with respect to σ_X, I obtain the following result:

[7] Note that $\frac{6k(\lambda - t)}{1\pm\sqrt{1-8k}} > \frac{-6kt}{1\pm\sqrt{1-8k}}$.

PROPOSITION 1: Unless the costs of compatibility are very large, when the firms are the same or close to the same size, the unilateral private incentives for each firm with respect to compatibility are too low. When the firms are not close in size, the smaller firm has socially excessive incentives to seek compatibility unilaterally.

The proposition is essentially consistent with the findings of Katz and Shapiro (1985) that firms with large networks or good reputations are biased against compatibility, whereas those with small networks or weak reputations are biased in favor of it.

IV. Equilibrium with Nonuser-Negative Effects

Now let us set parameters to the values proposed for case (II). (1) and (2) become:

$$U_A(j) = v + \theta - t(1-j) - p_A \tag{20}$$

$$U_B(j) = v - \theta - tj - \lambda Q_A - p_B \tag{21}$$

Again assume v large enough that all consumers choose A or B at equilibrium prices.[8] Combining (20) and (21) yields

$$\Psi_j = 2\theta + \lambda Q_A - t(1-2j) + p_B \tag{22}$$

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[8] As shall be shown, $Q_A + Q_B = 1$ implies v does not appear in the first-order conditions for A's profit maximization. This means $\lambda^*\big|_{Q_A+Q_B=1}$, A's profit-maximizing choice of λ subject to all consumers choosing to purchase A or B, is a function of exogenous parameters other than v. Accordingly, $\overline{v} \equiv \theta + t + \lambda^*\big|_{Q_A+Q_B=1} + \varepsilon$ satisfies the requirement for arbitrarily small $\varepsilon > 0$.

as the consumer's reservation price for A relative to B. Note that if $\lambda > 0$ the consumer's relative reservation price for A increases with Q_A; this reveals that a negative externality that selectively affects nonusers fosters a network externality.

Assume firm A incurs a fixed cost of *incompatibility*, $C(\lambda)$, which depends upon the size of the incompatibility cost imposed on product B's users by each incremental user of A. For simplicity, let us posit $C(\lambda) = k\lambda$ for $k, \lambda > 0$.[9] Firm A therefore sets p_A and λ to maximize

$$\Pi_A = p_A Q_A - k\lambda \tag{23}$$

while, as in the previous case, firm B sets p_B to maximize (7).

Assuming an interior solution, using (22), and making appropriate substitutions,

$$Q_A = \frac{t - p_A + p_B + 2\theta}{2t - \lambda} \tag{24}$$

and

$$Q_B = 1 - Q_A = \frac{t - \lambda + p_A - p_B - 2\theta}{2t - \lambda} \tag{25}$$

The first-order conditions for A's and B's profit maximization, respectively, are given by

$$\frac{t - 2p_A^* + p_B^* + 2\theta}{2t - \lambda^*} = 0 \tag{26}$$

$$\frac{p_A^* Q_A}{2t - \lambda^*} = k \tag{27}$$

and

[9] Nagler (2011) assumes a convex cost of incompatibility, with a linear, increasing marginal cost to enlarging the negative externality. The structure used here simplifies the equilibrium solution, but does not have a significant impact on the main results.

$$\frac{t-\lambda^* + p_A{}^* - 2p_B{}^* - 2\theta}{2t-\lambda^*} = 0 \tag{28}$$

Solving (26) and (28) together yields

$$p_A{}^* = t + \tfrac{2}{3}\theta - \tfrac{1}{3}\lambda^* \tag{29}$$

$$p_B{}^* = t - \tfrac{2}{3}\theta - \tfrac{2}{3}\lambda^* \tag{30}$$

Comparing (29) and (30) to (13) and (14), one is struck by the similarity of the equations. With σ_X set to zero, the equations are identical, but for the coefficients on λ^*. Thus, in the current case, we obtain a pricing result that is the precise flipside to the result in the previous case: *in*compatibility implies *lower* prices. In both the user-positive and nonuser-negative cases, the price effect is proportional to the size of the network effect.

However, if one compares the price differential in the current case with the differential in the previous case, an important difference emerges. With user-positive effects, the price differential between the products is independent of the network effect. This follows naturally from the mutuality of the effect. But with nonuser-negative effects, the price premium for product A *increases* with the network effect. Because this case involves a negative externality imposed unidirectionally, the "victimized" product, B, is in effect degraded relative to imposing product A.

Turning to the determination of equilibrium outcomes, substitution of (29) and (30) into (24) obtains

$$Q_A = \frac{t + \frac{2}{3}\theta - \frac{1}{3}\lambda^*}{2t-\lambda^*} \tag{31}$$

and substituting this into (27) yields

$$\lambda^* = 2t - \frac{\frac{1}{3}(t+2\theta)}{\sqrt{k} - \frac{1}{3}} \tag{32}$$

Note that an interior solution in quantities requires $\lambda < 2t$, hence $k < \frac{1}{9}$ for $\theta < -\frac{t}{2}$, and $k > \frac{1}{9}$ for $\theta > -\frac{t}{2}$. Moreover, $\lambda > 0$ requires $\frac{t+2\theta}{\sqrt{k}-\frac{1}{3}} < 6t$. Thus, observing what happens as *k* approaches $\frac{1}{9}$ in (31), it becomes evident that $k > \frac{1}{9}$ implies a corner solution of $\lambda = 0$ for all $\theta < -\frac{t}{2}$, and $k < \frac{1}{9}$ implies $\lambda = 0$ for all $\theta > -\frac{t}{2}$.

Does λ^* given in (32) represent a maximum? Using (29) and (31), we may re-write the first derivative of A's profit function with respect to λ as

$$\frac{\partial \Pi_A}{\partial \lambda} = Q_A^{\ 2} - k \tag{33}$$

The second derivative is therefore

$$\frac{\partial^2 \Pi_A}{\partial \lambda^2} = 2Q_A \frac{\partial Q_A}{\partial \lambda} \tag{34}$$

where, using (31),

$$\frac{\partial Q_A}{\partial \lambda} = \frac{\frac{1}{3}(t+2\theta)}{(2t-\lambda^*)^2} \tag{35}$$

Successive substitution of (32) into (35) and then into (34) shows that the second derivative is positive when $\theta > -\frac{t}{2}$, and negative otherwise. When demand for product A is relatively large, the marginal revenue product of λ increases in λ, while marginal cost of λ is constant. A corner solution equilibrium is the result: Firm A's profits are maximized by setting λ large enough to achieve $Q_A = 1$ (if the cost of increasing λ to this value is small enough relative to the benefit of taking the entire market) or else setting $\lambda = 0$ (if raising λ is prohibitively costly). However, when demand for product

A is relatively small, the marginal revenue product of λ decreases in λ, while marginal cost is constant. Consequently, the first-order condition for profit maximization yields a maximum.

For $\theta < -\frac{t}{2}$, the profit-maximum represented by (31) corresponds to the following prices and quantities:

$$\left(p_A^*, p_B^*\right) = \left(\frac{\frac{\sqrt{k}}{3}\left(t+2\theta\right)}{\sqrt{k}-\frac{1}{3}}, \frac{\frac{1}{3}\left(t+2\theta\right)\left[1-\sqrt{k}\right]}{\sqrt{k}-\frac{1}{3}} \right) \tag{36}$$

$$\left(Q_A^*, Q_B^*\right) = \left(\sqrt{k}, 1-\sqrt{k}\right) \tag{37}$$

As in the case of user-positive effects, I obtain a "buffering" result, that is, Firm A sets λ as a buffer to keep Q_A at an optimizing level that is independent of θ. A lower level of θ causes A to set λ lower and p_A higher. Firm B raises p_B as well – recall that prices *fall* with *in*compatibility – and the price differential $p_B - p_A$, which is positive in this region of low relative demand for A, increases as λ falls. Thus, Q_A remains steady at the level given in (37).

When demand is low enough to correspond to $\lambda = 0$ in (31), to wit, when $\theta < 3t\sqrt{k} - \frac{3}{2}t$, equilibrium prices and quantities are given, respectively, by

$$\left(p_A^*, p_B^*\right) = \left(t + \tfrac{2}{3}\theta; t - \tfrac{2}{3}\theta\right) \tag{38}$$

$$\left(Q_A, Q_B\right) = \left(\tfrac{1}{2} + \tfrac{\theta}{3t}; \tfrac{1}{2} - \tfrac{\theta}{3t}\right) \tag{39}$$

In this region, firm A favors full compatibility and allows Q_A to vary positively with p_A.

I now turn to the question of how the level of incompatibility chosen by firm A relates to the social optimum. Define welfare as above in (19). Substituting and integrating, I obtain

$$W = \Pi_A + [\lambda - t]Q_A^{\ 2} + [2\theta - p_A + t - \lambda]Q_A - \tfrac{t}{2} + v - \theta \qquad (40)$$

Differentiating with respect to λ, we obtain the following result:

PROPOSITION 2: When the imposing firm is small relative to its competitor, or when it is relatively large and the costs of incompatibility are large but not prohibitive, its incentives for incompatibility may be too low. When the imposing firm and its competitor are close in size, its incentives for incompatibility are too high, except when the costs of incompatibility are relatively large, in which case social and private incentives conform for zero incompatibility (i.e., perfect compatibility).

Table 1 summarizes more specifically the social optimality outcomes with respect to firm A's incompatibility decision in terms of the incompatibility cost parameter, *k*, and relative demand parameter, θ. As with Proposition 2, these results are derived in the appendix.

*** INSERT TABLE 1 APPROXIMATELY HERE ***

The intuition of the results for nonuser-negative effects can be seen from the car and sport-utility vehicle case example. When demands for cars and SUVs are relatively close in size, the SUV manufacturer's incentives for incompatibility may be excessive. Making SUVs more hazardous to car drivers provides maximum benefit to the SUV manufacturer when the network sizes for the two vehicle types are near equal because the effect on SUV sales at the margin is greatest. However, the social cost of vehicle incompatibility is also highest in this situation, since the probability of deadly car versus

SUV accidents is greatest when cars and SUVs coexist on the road in near equal numbers (White, 2004).

Meanwhile, when SUVs significantly outnumber cars, the manufacturer's incentives for incompatibility may be too low. This is because manufacturers fail to account for the social benefit that SUV-imposed external costs have of increasing homogeneity of the product mix, so that the incidence of car versus SUV accidents is reduced. Similarly, SUV firms' incentives for incompatibility are too low when cars significantly outnumber SUVs. In this situation, the increase in the price differential between SUVs and cars has a negative effect on SUV sales that outstrips the positive network effect. So, though SUVs are made more dangerous, the number of SUVs declines sufficiently to increase welfare overall. In both cases of lopsided network size, the manufacturer considers mainly the marginal effect of incompatibility on his sales, and this is smaller the more lopsided the network sizes are.

Though not exact, there is a strong correspondence between the results we obtained with respect to user-positive effects and those that arise under nonuser-negative effects. The clearest correspondence exists for firms with relatively low demand (i.e., small networks). I observe under nonuser-negative effects that such firms have suboptimal incentives for incompatibility from a social welfare perspective, just as firms with small networks had excessive incentives for compatibility under user-positive effects. When the two firms are close in size, the results also conform in most cases. When $k < \frac{2\sqrt{7}+1}{27}$ and $\theta \in \left(\frac{9tk}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$, firm A sets λ too high. Thus, under nonuser-negative effects, a firm's incentives for incompatibility may be excessive for moderate levels of relative demand, so long as the costs of incompatibility are not too large. This corresponds to the case of moderate demand under user-positive effects, in which private incentives for compatibility are too low.

Interestingly, with respect to firms with large networks, my results for the nonuser-negative case differ from Katz and Shapiro's (1985) findings for the user-positive case. While Katz and Shapiro find that firms with large networks or good reputations tend to be biased against compatibility, I find that they might be biased against *in*compatibility. Specifically, for $k \in \left(\frac{2\sqrt{7}+1}{27}, \frac{1}{3}\right)$, when $\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{9tk}{2}\right)$, firm A sets λ too low. The same thing happens for $k \in \left(\frac{1}{3}, \frac{2}{3}\right)$ when $\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{3t}{2}\right)$.

V. Conclusion

Previous analyses of incentives for compatibility in the context of network effects have focused on the case of user-positive effects. By and large, the results of these studies have suggested that firms focus primarily on compatibility as a tool to win over marginal customers, and they tend correspondingly to undervalue the utility that inframarginal customers gain from having a product that is compatible with products used by others. Thus, firms with large or moderate market shares, who have therefore a greater ratio of inframarginal to marginal consumers, tend to undervalue compatibility. Meanwhile, firms with small market shares place too much emphasis on it.

This paper has shown that a similar pattern of compatibility preferences relative to the social optimum exists for small and mid-sized firms under nonuser-negative effects. As in the user-positive case, the result relates to firms' incentives to win consumers at the margin; however, because the mechanism of the network effect is different in the nonuser-negative case, so is the logic of the result. Nonuser-negative effects result from negative externalities that users impose on nonusers, thus both their value to the imposing firm and their adverse social effects are stronger the more "contact points" there are between users and nonusers. For this reason, firms' incentives for incompatibility tend to be excessive when market shares are near-equal. Correspondingly, when a firm has a small market share, the number of contact points with nonusers is diminished because the firm has a smaller installed base. This decreases its incentives for incompatibility. Meanwhile, the adverse social effects of incompatibility are also decreased, while the social benefit that increased incompatibility has through its ability to shift consumers and increase homogeneity in the product mix becomes relatively prominent. The result is that a firm's incentives for incompatibility may be too low when its market share is small.

The paper has further indicated that, with respect to firms with large market shares, the social optimality of firms' compatibility incentives may differ in the nonuser-negative case relative to the user-positive case. Indeed, the same mechanism is at work for large and small firms under nonuser-negative effects: private benefits to incompatibility are diminished when firm sizes are lopsided, but social benefits are increased. This represents a difference relative to the conventional, user-positive case.

The general implication is that public policy has a role in encouraging compatibility when competing products have near-equal network sizes. This is true not only in the case of user-positive effects, but also when external costs are imposed selectively by users on non-users. Conversely, policy makers may need to dampen unilateral private incentives for compatibility at the margin when network sizes are lopsided. The surprising thing is that this may actually mean encouraging firms to impose larger external costs that selectively affect rivals' products. For example, if SUVs represented a small enough share of the motor vehicle market, it might actually improve welfare to make them more hazardous to car drivers, because the price effects of doing would further curtail sales of SUVs. If instead the overwhelming majority of vehicles were SUVs, making them more hazardous would again improve welfare – in this case, by reducing further the number of car drivers that incur incompatibility losses due to SUVs. In both situations, increased incompatibility at a per-unit level improves welfare by increasing standardization and thereby reducing the adverse effects of incompatibility at an aggregate level.

Beyond pure compatibility considerations, the broader implications of my results for public policy are perhaps equally surprising. The wisdom that external costs are provided excessively in the market and should be reduced is called into question when one considers that, in many cases, such costs have implications for the competitive equilibrium in markets.[10] Situations involving user-imposed externalities should be scrutinized to consider whether the externalities selectively, or asymmetrically, affect non-users (i.e., are nonuser-negative). The desirability of certain policy prescriptions, such as the use of Pigouvian taxes, might be affected by such asymmetries.

Appendix

A1. Second Order Conditions – Positive Consumption Externalities Case

The Hessian in this case is given by

[10] This issue is explored directly by Nagler (2011).

$$|H| = \begin{vmatrix} \frac{\partial^2 \Pi_A}{\partial p_A{}^2} & \frac{\partial^2 \Pi_A}{\partial p_A \partial \sigma_X} \\ \frac{\partial^2 \Pi_A}{\partial p_A \partial \sigma_X} & \frac{\partial^2 \Pi_A}{\partial \sigma_X{}^2} \end{vmatrix} \tag{41}$$

where, using (8), (13), (14), and the first-order condition $Q_A + p_A \frac{\partial Q_A}{\partial p_A} = 0$, the components are given by

$$\frac{\partial^2 \Pi_A}{\partial \sigma_X{}^2} = p_A \frac{\partial^2 Q_A}{\partial \sigma_X{}^2} = p_A \frac{\partial}{\partial \sigma_X} \frac{2\left[t-(1-\sigma_X)\lambda\right]\lambda - 2\lambda\left[t - p_A + p_B - (1-\sigma_X)\lambda + 2\theta\right]}{4\left[t-(1-\sigma_X)\lambda\right]^2}$$
$$= p_A \frac{2\lambda^2\left\{\frac{2}{3}\theta\right\}}{2\left[t-(1-\sigma_X)\lambda\right]^3} \tag{42}$$

$$\frac{\partial^2 \Pi_A}{\partial p_A{}^2} = 2\frac{\partial Q_A}{\partial p_A} + p\frac{\partial^2 Q_A}{\partial p_A{}^2} = 2\left[\frac{-1}{2\left[t-(1-\sigma_X)\lambda\right]}\right] + p[0] = \frac{-1}{\left[t-(1-\sigma_X)\lambda\right]} \tag{43}$$

$$\frac{\partial^2 \Pi_A}{\partial p_A \partial \sigma_X} = \frac{\partial Q_A}{\partial \sigma_X} + p_A \frac{\partial^2 Q_A}{\partial p_A \partial \sigma_X} = \frac{\lambda\left(\frac{1}{2} - Q_A\right)}{\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]} + p_A \frac{-\lambda \frac{\partial Q_A}{\partial p_A}}{\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]} = \frac{\lambda}{2\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]} \tag{44}$$

Substitution into (41) yields

$$|H| = \begin{vmatrix} \frac{-1}{\left[t-(1-\sigma_X)\lambda\right]} & \frac{\lambda}{2\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]} \\ \frac{\lambda}{2\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]} & p_A \frac{2\lambda^2\left\{\frac{2}{3}\theta\right\}}{2\left[t-(1-\sigma_X)\lambda\right]^3} \end{vmatrix}$$
$$= \frac{-\lambda^2\left\{\frac{4}{3}\theta + t - \left(1-\sigma_X{}^*\right)\lambda\right\}^2}{4\left[t-(1-\sigma_X)\lambda\right]^4} < 0 \tag{45}$$

So all solutions to the first-order conditions are maxima.

A2. Proof of Proposition 1

Making substitutions from the model in (19) and integrating, we obtain

$$W = \Pi_A + \left[2(1-\sigma_X)\lambda - t\right]Q_A{}^2 + \left(2\theta + t + 2\sigma_X\lambda - p_A - 2\lambda\right)Q_A + v - \theta + \lambda - \tfrac{t}{2} \tag{46}$$

Differentiate (46) with respect to σ_X, use (13) and (15), and assume an interior solution:

$$
\begin{aligned}
\frac{\partial W}{\partial \sigma_X} &= \frac{\partial \Pi_A}{\partial \sigma_X} + \frac{\partial}{\partial\left[\left(1-\sigma_X\right)\lambda\right]}\left\{\begin{array}{l}\left[2\left(1-\sigma_X\right)\lambda - t\right]Q_A^2 \\ +\left(2\theta - p_A - \left[2\left(1-\sigma_X\right)\lambda - t\right]\right)Q_A\end{array}\right\}\cdot\frac{\partial\left[\left(1-\sigma_X\right)\lambda\right]}{\partial \sigma_X} \\
&= 0 + \left\{\begin{array}{l}2Q_A^2 + 2\left[2\left(1-\sigma_X\right)\lambda - t\right]Q_A \frac{\partial Q_A}{\partial\left[\left(1-\sigma_X\right)\lambda\right]} \\ -\left[2+\frac{\partial p_A}{\partial\left[\left(1-\sigma_X\right)\lambda\right]}\right]Q_A + \left(2\theta - p_A - \left[2\left(1-\sigma_X\right)\lambda - t\right]\right)\frac{\partial Q_A}{\partial\left[\left(1-\sigma_X\right)\lambda\right]}\end{array}\right\}\cdot -\lambda \\
&= \left\{\begin{array}{l}2Q_A^2 + 2\left[2\left(1-\sigma_X\right)\lambda - t\right]Q_A\left[\frac{-2\left[t-\left(1-\sigma_X\right)\lambda\right]+2p_A}{4\left[t-\left(1-\sigma_X\right)\lambda\right]^2}\right] \\ -\left[2+-1\right]Q_A + \left(2\theta - p_A - \left[2\left(1-\sigma_X\right)\lambda - t\right]\right)\left[\frac{-2\left[t-\left(1-\sigma_X\right)\lambda\right]+2p_A}{4\left[t-\left(1-\sigma_X\right)\lambda\right]^2}\right]\end{array}\right\}\cdot -\lambda \\
&= \left\{2Q_A^2 - Q_A + \left\{\left(2Q_A - 1\right)\left[2\left(1-\sigma_X\right)\lambda - t\right] + 2\theta - p_A\right\}\left[\frac{-1}{2\left[t-\left(1-\sigma_X\right)\lambda\right]} + \frac{p_A}{2\left[t-\left(1-\sigma_X\right)\lambda\right]^2}\right]\right\}\cdot -\lambda \\
&= \left\{2Q_A\left(Q_A - \tfrac{1}{2}\right) + \frac{\left\{\left(2Q_A - 1\right)\left[2\left(1-\sigma_X\right)\lambda - t\right] + 2\theta - p_A\right\}\left(Q_A - \tfrac{1}{2}\right)}{\left[t-\left(1-\sigma_X\right)\lambda\right]}\right\}\cdot -\lambda \\
&= -\lambda\left(Q_A - \tfrac{1}{2}\right)\left\{2Q_A + \frac{2Q_A\left[2\left(1-\sigma_X\right)\lambda - t\right]}{t-\left(1-\sigma_X\right)\lambda} + \frac{2\theta - p_A - 2\left(1-\sigma_X\right)\lambda + t}{t-\left(1-\sigma_X\right)\lambda}\right\} \qquad (47) \\
&= -\lambda\left(Q_A - \tfrac{1}{2}\right)\left\{\frac{2Q_A\left(1-\sigma_X\right)\lambda}{t-\left(1-\sigma_X\right)\lambda} + \frac{2\theta - p_A - 2\left(1-\sigma_X\right)\lambda + t}{t-\left(1-\sigma_X\right)\lambda}\right\} \\
&= -\lambda\left(Q_A - \tfrac{1}{2}\right)\left\{\frac{2\left\{\frac{t+\frac{2}{3}\theta-\left(1-\sigma_X^*\right)\lambda}{2\left[t-\left(1-\sigma_X^*\right)\lambda\right]}\right\}\left(1-\sigma_X^*\right)\lambda}{t-\left(1-\sigma_X^*\right)\lambda} + \frac{2\theta - \left[t+\frac{2}{3}\theta-\left(1-\sigma_X^*\right)\lambda\right] - 2\left(1-\sigma_X^*\right)\lambda + t}{t-\left(1-\sigma_X^*\right)\lambda}\right\} \\
&= -\lambda\left(Q_A - \tfrac{1}{2}\right)\left\{\frac{2\left(1-\sigma_X^*\right)\lambda\left\{t+\frac{2}{3}\theta-\left(1-\sigma_X^*\right)\lambda\right\}}{2\left[t-\left(1-\sigma_X^*\right)\lambda\right]^2} + \frac{\frac{4}{3}\theta-\left(1-\sigma_X^*\right)\lambda}{t-\left(1-\sigma_X^*\right)\lambda}\right\} \\
&= -\lambda\left(Q_A - \tfrac{1}{2}\right)\left\{\frac{\frac{4}{3}\theta\left(1-\sigma_X^*\right)\lambda}{2\left[t-\left(1-\sigma_X^*\right)\lambda\right]^2} + \frac{\frac{4}{3}\theta}{\left[t-\left(1-\sigma_X^*\right)\lambda\right]}\right\} \\
&= -\frac{\frac{4}{3}\theta\lambda\left(Q_A - \frac{1}{2}\right)}{2\left[t-\left(1-\sigma_X^*\right)\lambda\right]^2}\left[2t-\left(1-\sigma_X^*\right)\lambda\right]
\end{aligned}
$$

Since an interior solution requires $\theta < 0$ and $Q_A < \frac{1}{2}$, it follows that $\frac{\partial W}{\partial \sigma_X} < 0$ for all interior solutions. Thus, whenever firm A's network size is small enough that it chooses at least partial compatibility, it overinvests in compatibility.

Now we consider the corner solution corresponding to $\sigma_X = 0$. We begin by noting that $\frac{\partial \Pi_A}{\partial \sigma_X} \neq 0$ at $\sigma_X = 0$; therefore we may substitute (47) in for $\frac{\partial W}{\partial \sigma_X}$, but we must add $\frac{\partial \Pi_A}{\partial \sigma_X}$ back in. We do so and evaluate the resulting expression at $\sigma_X = 0$:

$$
\begin{aligned}
\frac{\partial W}{\partial \sigma_X} &= p_A \frac{\partial Q_A}{\partial \sigma_X} + Q_A \frac{\partial p_A}{\partial \sigma_X} - k\lambda - \frac{\frac{4}{3}\theta\lambda\left(Q_A - \frac{1}{2}\right)}{2\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]^2}\left[2t-\left(1-\sigma_X{}^*\right)\lambda\right] \\
&= -\lambda p_A \frac{-2\left[t-\left(1-\sigma_X\right)\lambda\right]+2p_A}{4\left[t-\left(1-\sigma_X\right)\lambda\right]^2} + Q_A\lambda - k\lambda - \frac{\frac{4}{3}\theta\lambda\left(Q_A-\frac{1}{2}\right)}{2\left[t-\left(1-\sigma_X{}^*\right)\lambda\right]^2}\left[2t-\left(1-\sigma_X{}^*\right)\lambda\right] \\
&= -\lambda p_A \frac{p_A - [t-\lambda]}{2[t-\lambda]^2} + \frac{p_A}{2(t-\lambda)}\lambda - k\lambda - \frac{\frac{4}{3}\theta\lambda\left(\frac{p_A}{2(t-\lambda)}-\frac{1}{2}\right)}{2[t-\lambda]^2}[2t-\lambda] \\
&= \frac{1}{2(t-\lambda)^2}\left[\lambda\left(t-\lambda+\tfrac{2}{3}\theta\right)\left(t-\lambda-\tfrac{2}{3}\theta\right) - 2k\lambda(t-\lambda)^2 - \frac{\frac{4}{9}\theta^2\lambda(2t-\lambda)}{(t-\lambda)}\right] \\
&= \frac{\lambda}{2(t-\lambda)^2}\left[(t-\lambda)^2 - \tfrac{4}{9}\theta^2 - 2k(t-\lambda)^2 - \frac{\frac{4}{9}\theta^2(2t-\lambda)}{(t-\lambda)}\right] \\
&= \frac{\lambda}{2(t-\lambda)^3}\left[-\tfrac{4}{9}\theta^2(t-\lambda) + (1-2k)(t-\lambda)^3 - \tfrac{4}{9}\theta^2(2t-\lambda)\right] \\
&= \frac{\lambda}{2(t-\lambda)^3}\left[(1-2k)(t-\lambda)^3 - \tfrac{4}{9}\theta^2(3t-2\lambda)\right]
\end{aligned}
\tag{48}
$$

So long as $k \le \frac{1}{2}$, $\frac{\partial W}{\partial \sigma_X} > 0$ for θ close to zero.

A3. Proof of Proposition 2 and Derivation of Table 1

To begin, let us differentiate (40) with respect to λ, assume an interior solution (i.e., $\theta \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$ and $k < \frac{1}{9}$),

$$W = \Pi_A + [\lambda - t]Q_A^{\,2} + [2\theta - p_A + t - \lambda]Q_A - \tfrac{t}{2} + v - \theta$$
$$\Rightarrow \tfrac{\partial W}{\partial \lambda} = 0 + 2[\lambda - t]Q_A \tfrac{\partial Q_A}{\partial \lambda} + Q_A^{\,2} - \left(\tfrac{\partial p_A}{\partial \lambda} + 1\right)Q_A + [2\theta - p_A + t - \lambda]\tfrac{\partial Q_A}{\partial \lambda} \tag{49}$$

Using (29) and (31), we find

$$\frac{\partial p_A}{\partial \lambda} = -\tfrac{1}{3}; \frac{\partial Q_A}{\partial \lambda} = \frac{Q_A - \tfrac{1}{3}}{2t - \lambda} \tag{50}$$

Substituting (29), (31), and (50) into (49) and factoring yields

$$\tfrac{\partial W}{\partial \lambda} = \frac{-\tfrac{1}{3}(2t-\lambda)^3 - \tfrac{4}{3}t(2\theta + t)(2t - \lambda) + \tfrac{1}{3}(4t - \lambda)(2\theta + t)^2}{3(2t-\lambda)^3} \tag{51}$$

Since $\lambda < 2t$ on $0 < Q_A < 1$, hence on c, it follows that $\tfrac{\partial W}{\partial \lambda} < 0$ at $\theta = -\tfrac{t}{2}$. If we can show that $\tfrac{\partial W}{\partial \lambda} > 0$ at $\theta = -\tfrac{3t}{2}$, then we will have proven that there exists $\underline{\theta}(k) \in \left(-\tfrac{3t}{2}, -\tfrac{t}{2}\right)$ such that, for $\theta < \underline{\theta}$, firm A sets λ too low. In the neighborhood of $\theta = -\tfrac{3t}{2}$, $\tfrac{\partial \Pi_A}{\partial \lambda}$ approaches $-k$. So, using (51), and substituting in $\theta = -\tfrac{3t}{2}$ and $\lambda = 0$, we obtain:

$$\begin{aligned}
\tfrac{\partial W}{\partial \lambda} &= \tfrac{\partial \Pi_A}{\partial \lambda} + \frac{-\tfrac{1}{3}(2t-\lambda)^3 - \tfrac{4}{3}t(2\theta + t)(2t - \lambda) + \tfrac{1}{3}(4t - \lambda)(2\theta + t)^2}{3(2t-\lambda)^3} \\
&= -k + \frac{-\tfrac{1}{3}(2t-\lambda)^3 - \tfrac{4}{3}t(2\theta + t)(2t - \lambda) + \tfrac{1}{3}(4t - \lambda)(2\theta + t)^2}{3(2t-\lambda)^3} \\
&= -k + \frac{-\tfrac{8}{3}t^3 - \tfrac{4}{3}t(-2t)(2t) + \tfrac{1}{3}(4t)(-2t)^2}{3(2t)^3} \\
&= -k + \frac{-\tfrac{8}{3}t^3 + \tfrac{16}{3}t^3 + \tfrac{16}{3}t^3}{24t^3} = -k + \frac{\tfrac{24}{3}t^3}{24t^3} = -k + \frac{1}{3}
\end{aligned} \tag{52}$$

So, $\tfrac{\partial W}{\partial \lambda} > 0$ if and only if $k < \tfrac{1}{3}$. This satisfies the interior solution requirement of $k < \tfrac{1}{9}$, so we have proven the first part for this case.

Now consider $\theta \in \left(-\tfrac{3t}{2}, -\tfrac{t}{2}\right)$ with $k \in \left(\tfrac{1}{9}, \tfrac{1}{3}\right)$. In this case, *k* is sufficiently large that a corner solution of $\lambda = 0$ holds for all $\theta < -\tfrac{t}{2}$. Note that $\tfrac{\partial \Pi_A}{\partial \lambda} \neq 0$ at $\lambda = 0$;

therefore we may substitute (52) in for $\frac{\partial W}{\partial \lambda}$, but we must add $\frac{\partial \Pi_A}{\partial \lambda}$ back in. We do so and evaluate the resulting expression at $\lambda = 0$, simplifying:

$$\frac{\partial W}{\partial \lambda} = p_A \frac{\partial Q_A}{\partial \lambda} + Q_A \frac{\partial p_A}{\partial \lambda} - k + \frac{-\frac{1}{3}(2t-\lambda)^3 - \frac{4}{3}t(2\theta+t)(2t-\lambda) + \frac{1}{3}(4t-\lambda)(2\theta+t)^2}{3(2t-\lambda)^3}$$

$$= p_A\left(\frac{Q_A - \frac{1}{3}}{2t-\lambda}\right) - \frac{1}{3}Q_A - k + \frac{-\frac{1}{3}(2t-\lambda)^3 - \frac{4}{3}t(2\theta+t)(2t-\lambda) + \frac{1}{3}(4t-\lambda)(2\theta+t)^2}{3(2t-\lambda)^3}$$

$$= \left(\frac{t + \frac{2}{3}\theta - \frac{1}{3}\lambda}{2t-\lambda}\right)^2 - \frac{2}{3}\left(\frac{t + \frac{2}{3}\theta - \frac{1}{3}\lambda}{2t-\lambda}\right) - k$$

$$+ \frac{-\frac{1}{3}(2t-\lambda)^3 - \frac{4}{3}t(2\theta+t)(2t-\lambda) + \frac{1}{3}(4t-\lambda)(2\theta+t)^2}{3(2t-\lambda)^3}$$

$$= \left(\frac{t + \frac{2}{3}\theta}{2t}\right)^2 - \frac{2}{3}\left(\frac{t + \frac{2}{3}\theta}{2t}\right) - k + \frac{-\frac{1}{3}(2t)^3 - \frac{4}{3}t(2\theta+t)(2t) + \frac{1}{3}(4t)(2\theta+t)^2}{3(2t)^3}$$

$$= \frac{6t\left(t + \frac{2}{3}\theta\right)^2}{3(2t)^3} - \frac{8t^2\left(t + \frac{2}{3}\theta\right)}{3(2t)^3} + \frac{-3k(2t)^3 - \frac{1}{3}(2t)^3 - \frac{4}{3}t(2\theta+t)(2t) + \frac{1}{3}(4t)(2\theta+t)^2}{3(2t)^3}$$

$$= \frac{\frac{8}{3}\theta t - 24kt^2 - 6t^2 + 8\theta^2}{24t^2} \tag{53}$$

Differentiating this expression with respect to θ reveals that $\frac{\partial W}{\partial \lambda}$ is monotone in θ on $\theta \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$:

$$\frac{\partial^2 W}{\partial \lambda \partial \theta} = \frac{\frac{8}{3}t + 16\theta}{24t^2} = 0 \Rightarrow \theta = -\frac{t}{6} > -\frac{t}{2} \tag{54}$$

It remains to check the sign of $\frac{\partial W}{\partial \lambda}$ at the endpoints of the interval. At $\theta = -\frac{3t}{2}$, $\frac{\partial W}{\partial \lambda} = \frac{1}{3} - k$, which is positive for $k < \frac{1}{3}$. At $\theta = -\frac{t}{2}$, it can shown using (53) that $\frac{\partial W}{\partial \lambda} < 0$. Therefore, for $k \in \left(\frac{1}{9}, \frac{1}{3}\right)$, there exists $\underline{\theta}(k) \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$ such that, for $\theta < \underline{\theta}$, firm A sets λ too low. (It may be observed in passing that $k > \frac{1}{3}$ implies $\frac{\partial W}{\partial \lambda} < 0$ everywhere on

$\theta \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$, so private and social incentives conform for setting $\lambda = 0$ when $\theta \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$ and $k > \frac{1}{3}$.)

Now consider $\theta \in \left(-\frac{t}{2}, \frac{3t}{2}\right)$. As noted in the text, this range corresponds to a corner solution in λ: firm A sets λ to achieve $Q_A = 1$ when the cost of increasing λ is small enough, and it sets $\lambda = 0$ otherwise. Using (31), we find that $\lambda = \frac{3}{2}t - \theta$ corresponds to $Q_A = 1$. Using (23) and comparing firm A's profits at $\lambda = 0$ with its profits when $Q_A = 1$ and $\lambda = \frac{3}{2}t - \theta$, we find that firm A will opt for $Q_A = 1$ when $k < \frac{2\theta}{9t}$, or, rearranging, when $\theta > \frac{9}{2}tk$. Thus, we are able to recast firm A's threshold in terms of a level of demand large enough to make increasing λ worthwhile.

The corresponding social threshold in θ for raising λ to set $Q_A = 1$ is derived by substituting $\lambda = 0$ into $\frac{\partial W}{\partial \lambda} = 0$ and solving for θ. Setting (53) equal to zero and solving for θ yields (using quadratic formula) $\theta = t\left[\frac{-1 \pm 2\sqrt{7+27k}}{6}\right]$. Here, only the positive-signed root corresponds to $\theta \in \left(-\frac{t}{2}, \frac{3t}{2}\right)$, so that is the relevant one. One can check that $\theta > t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]$ corresponds to $\frac{\partial W}{\partial \lambda} > 0$ in (53).

Equating the social and private thresholds and solving for k:

$$t\left[\frac{-1+2\sqrt{7+27k}}{6}\right] = \frac{9tk}{2} \Rightarrow k = \frac{1 \pm 2\sqrt{7}}{27} \qquad (55)$$

where only the positive-signed root corresponds to $k > 0$ and is therefore relevant. Thus, for $k > \frac{1+2\sqrt{7}}{27}$, the private threshold level of θ exceeds the social threshold, so that for $\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{3t}{2}\right)$, firm A sets λ too low, while for $\theta \in \left(-\frac{t}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$, firm A's incentives conform with social incentives for setting $\lambda = 0$. Meanwhile, for $k < \frac{1+2\sqrt{7}}{27}$, the social threshold exceeds the private threshold, so that for $\theta \in \left(\frac{9tk}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$, firm A sets λ too high. When $\theta \in \left(-\frac{t}{2}, \frac{9tk}{2}\right)$, $k < \frac{1+2\sqrt{7}}{27}$ corresponds to firm A's incentives conforming with social incentives for setting $\lambda = 0$. Finally, $\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{3t}{2}\right)$ and $k < \frac{1+2\sqrt{7}}{27}$ imply that private and social incentives conform for setting $\lambda = \frac{3}{2}t - \theta$ and $Q_A = 1$.

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Table 1. Summary of social optimality outcomes for firm A's incompatibility decision (nonuser-negative case).

<table>
<tr><th></th><th>$\theta < -\frac{3t}{2}$</th><th>$\theta \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$</th><th>$\theta \in \left(-\frac{t}{2}, \frac{3t}{2}\right)$</th><th>$\theta > \frac{3t}{2}$</th></tr>
<tr><td>$k \in \left(0, \frac{1+2\sqrt{7}}{27}\right)$</td><td rowspan="4">Private and social incentives conform for $\lambda = 0$.</td><td rowspan="2">There exists $\underline{\theta}(k) \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$ such that for $\theta < \underline{\theta}$ firm A sets λ too low.</td><td>$\theta \in \left(-\frac{t}{2}, \frac{9tk}{2}\right)$: private and social incentives conform for $\lambda = 0$.
$\theta \in \left(\frac{9tk}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$: firm A sets λ too high.
$\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{3t}{2}\right)$: private and social incentives conform for $Q_A = 1$.</td><td rowspan="4">Private and social incentives conform for $\lambda = 0$. ($Q_A = 1$ regardless.)</td></tr>
<tr><td>$k \in \left(\frac{1+2\sqrt{7}}{27}, \frac{1}{3}\right)$</td><td>$\theta \in \left(-\frac{t}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$: private and social incentives conform for $\lambda = 0$.
$\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{9tk}{2}\right)$: firm A sets λ too low.
$\theta \in \left(\frac{9tk}{2}, \frac{3t}{2}\right)$: private and social incentives conform for $Q_A = 1$.</td></tr>
<tr><td>$k \in \left(\frac{1}{3}, \frac{2}{3}\right)$</td><td rowspan="2">Private and social incentives conform for $\lambda = 0$.</td><td>$\theta \in \left(-\frac{t}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$: private and social incentives conform for $\lambda = 0$.
$\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{3t}{2}\right)$: firm A sets λ too low.</td></tr>
<tr><td>$k > \frac{2}{3}$</td><td>Private and social incentives conform for $\lambda = 0$.</td></tr>
</table>